DEFINITION

A number of the form $x + iy$, where x and y are real numbers and $i = \sqrt{-1}$ is called a complex number. It is usually denoted by z, *i.e.*, $z = x + iy$

The real and imaginary parts of a complex number z denoted by Re(z) and Im(z) respectively. If $z = x + iy$, then Re(z) = x and Im(z) = y.

- A complex number z is said to be
  (i) Purely real, if Im(z) = 0
  (ii) Purely imaginary, if Re(z) = 0

Note : Order relations, "greater than" and "less than" are not defined for complex numbers.

EQUALITY OF COMPLEX NUMBERS

Two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are said to be equal if and only if $x_1 = x_2$ and $y_1 = y_2$
i.e., $z_1 = z_2 \Leftrightarrow \text{Re}(z_1) = \text{Re}(z_2)$ and $\text{Im}(z_1) = \text{Im}(z_2)$

ALGEBRA OF COMPLEX NUMBERS

1. Addition of two Complex Numbers

Let $z_1 = x + iy$ and $z_2 = u + iv$ be two complex numbers. Then the sum $z_1 + z_2 = (x + u) + i(y + v)$

Properties	
Closure law	$z_1 + z_2$ is a complex number for all complex numbers z_1 and z_2.
Commutative law	$z_1 + z_2 = z_2 + z_1$, where z_1 and z_2 are complex numbers.
Associative law	$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$, where z_1, z_2 and z_3 are complex numbers.
Existence of additive identity	There exists a complex number $0 + i0$ (denoted as 0) is called additive identity.
Existence of additive inverse	For every complex number $z = x + iy$, we have $-x + i(-y)$ (denoted as $-z$) is called additive inverse or negative of z.

2. Difference of two Complex Numbers

For two complex numbers z_1 and z_2, $z_1 - z_2 = z_1 + (-z_2)$

3. Multiplication of two Complex Numbers

Let $z_1 = x + iy$ and $z_2 = u + iv$ be any two complex numbers. Then the product z_1z_2 is defined as $z_1z_2 = (xu - yv) + i(xv + yu)$

Properties	
Closure law	The product of two complex numbers *i.e.*, $z_1\, z_2$ is a complex number, where z_1 and z_2 are complex numbers.
Commutative law	For any two complex numbers z_1 and z_2, we have $z_1 z_2 = z_2 z_1$
Associative law	For any three complex numbers z_1, z_2, z_3, we have $(z_1 z_2)z_3 = z_1(z_2 z_3)$
Existence of multiplicative identity	There exists the complex number $1 + i0$ (denoted as 1), called the multiplicative identity such that $z \cdot 1 = z$, for every complex number z.
Existence of multiplicative inverse	For every non-zero complex number $z = x + iy$ or $x + yi (x \neq 0, y \neq 0)$, we have the complex number $\frac{x}{x^2+y^2} + i\frac{-y}{x^2+y^2}$ (denoted by $\frac{1}{z}$ or z^{-1}), called the multiplicative inverse of z such that $z \cdot \frac{1}{z} = 1$ (the multiplicative identity)
Distributive law	For any three complex numbers z_1, z_2, z_3, we have (a) $z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$ (b) $(z_1 + z_2)z_3 = z_1 z_3 + z_2 z_3$

4. Division of two Complex Numbers

For two complex numbers z_1 and z_2, where $z_2 \neq 0$, the quotient $\frac{z_1}{z_2}$ is defined by $\frac{z_1}{z_2} = z_1 \frac{1}{z_2}$.

IDENTITIES OF COMPLEX NUMBERS

If z_1 and z_2 are two complex numbers, then

- $(z_1 + z_2)^2 = z_1^2 + 2z_1 z_2 + z_2^2$
- $(z_1 - z_2)^2 = z_1^2 - 2z_1 z_2 + z_2^2$
- $(z_1 + z_2)^3 = z_1^3 + 3z_1^2 z_2 + 3z_1 z_2^2 + z_2^3$
- $(z_1 - z_2)^3 = z_1^3 - 3z_1^2 z_2 + 3z_1 z_2^2 - z_2^3$
- $(z_1^2 - z_2^2) = (z_1 + z_2)(z_1 - z_2)$

MODULUS AND CONJUGATE OF A COMPLEX NUMBER

Modulus of a complex number $z = x + iy$ is denoted by $|z|$ and it is defined as $|z| = \sqrt{x^2 + y^2}$

Conjugate of a complex number $z = x + iy$ is denoted by $\bar{z}$ and it is defined as $\bar{z} = x - iy$.

PROPERTIES OF MODULUS AND CONJUGATE OF A COMPLEX NUMBER

- $|z_1 z_2| = |z_1|\,|z_2|$
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, provided $|z_2| \neq 0$
- $\overline{(z_1 \cdot z_2)} = (\bar{z}_1)\,(\bar{z}_2)$
- $\overline{(z_1 \pm z_2)} = \bar{z}_1 \pm \bar{z}_2$
- $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}, (\bar{z}_2 \neq 0)$

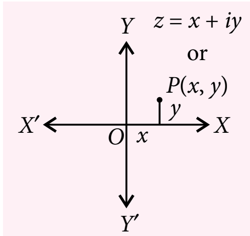
MULTIPLICATIVE INVERSE

Let $z = x + iy$ be a non-zero complex number. Then,

$$\frac{1}{z} = \frac{1}{x+iy} = \frac{x-iy}{x^2+y^2} = \frac{\bar{z}}{|z|^2} \Rightarrow z^{-1} = \frac{\bar{z}}{|z|^2}$$

ARGAND PLANE AND GEOMETRICAL REPRESENTATION OF COMPLEX NUMBERS

Let O be the origin and OX and OY be the x-axis and y-axis, respectively. Then, any complex number $z = x + iy = (x, y)$ may be represented by a unique point P whose coordinates are (x, y).



The plane on which complex numbers are represented is known as the Complex plane or Argand's plane.

POLAR FORM OF A COMPLEX NUMBER

Let O be the origin and OX and OY be the x-axis and y-axis respectively. Let $z = x + iy$ be a complex number represented by the point $P(x, y)$.

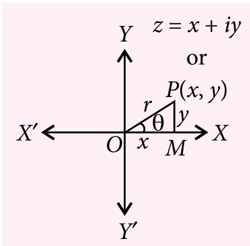
Draw $PM \perp OX$ as shown in given figure. Then, $OM = x$ and $PM = y$. Join OP. Let $OP = r$ and $\angle XOP = \theta$. Then $x = r\cos\theta$ and $y = r\sin\theta$.

$\therefore \quad z = x + iy = r(\cos\theta + i\sin\theta)$

$\Rightarrow \quad r = \sqrt{x^2 + y^2} = |z|$

Also $\quad \tan\theta = \frac{y}{x}$

This form $z = r(\cos\theta + i\sin\theta) = re^{i\theta}$ is called polar form of the complex number z. Angle θ is known as amplitude or agrument of z, written as $\arg(z)$.



The unique value of θ such that $-\pi < \theta \le \pi$ for which $x = r\cos\theta$ and $y = r\sin\theta$, is known as the principal value of the argument.

The general value of the argument is $(2n\pi + \theta)$, where n is an integer and θ is the principal value of $\arg(z)$.

PROPERTIES OF ARGUMENT OF COMPLEX NUMBERS

(i) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) + 2k\pi$ $(k = 0 \text{ or } 1 \text{ or } -1)$

(ii) $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) + 2k\pi$ $(k = 0 \text{ or } 1 \text{ or } -1)$

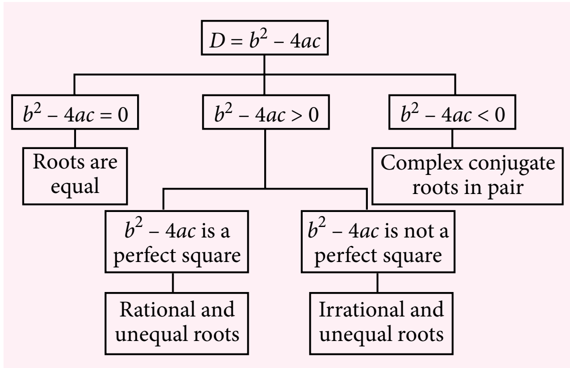
(iii) $\arg(z^n) = n\arg(z) + 2k\pi$ $(k = 0 \text{ or } 1 \text{ or } -1)$

(iv) $\arg(\bar{z}) = -\arg(z)$

QUADRATIC EQUATIONS

An equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \ne 0$ is known as quadratic equation. Then the solutions of the given equation is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{4ac - b^2}\,i}{2a}$$



LINEAR INEQUALITIES

DEFINITION

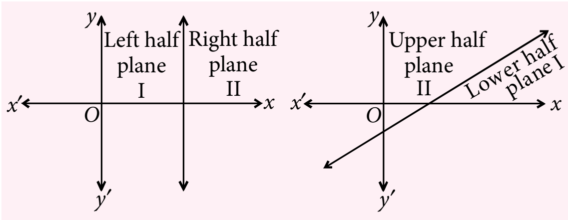
A statement involving variable(s) and the sign of inequality *i.e.*, >, <, ≥ or ≤ is called an inequation or an inequality.

	Inequalities	Definition	Examples
1.	Numerical	Inequalities which do not involve variables are called numerical inequalities.	$3 < 7$, $6 > 5$
2.	Literal	Inequalities which involve variables are called literal inequalities.	$x \ge 4$, $y > 6$, $x - y < 0$
3.	Linear	An equation of the form $ax + b < 0$, or $ax + b \ge 0$, or $ax + by > 0$ etc. are known as linear inequations.	$x + 3 < 0$, $3x + 2y > 7$
4.	Quadratic	An equation of the form $ax^2 + bx + c < 0$, or $ax^2 + bx + c \le 0$, or $ax^2 + bx + c > 0$, or $ax^2 + bx + c \ge 0$ is known as a quadratic inequation.	$x^2 + 7x + 4 > 0$
5.	Strict	Inequalities involving the symbols '>' or '<' are called strict inequalities.	$x + y > 5$, $y < 0$
6.	Slack	Inequalities involving the symbols '≥' or '≤' are called slack inequalities.	$4x + 3y \ge 2$, $x \le -4$

RULES FOR SOLVING INEQUALITIES

- Same number or expression may be added to or subtracted from both sides of an inequality without affecting the sign of inequality.
- Both sides of an inequality can be multiplied or divided by the same positive number without affecting the inequality sign.
- When both sides of an inequation is multiplied or divided by a negative number, then sign of inequality changes.

GRAPHICAL SOLUTION OF LINEAR INEQUALITIES IN TWO VARIABLES



The region containing all the solutions of an inequality is called the solution region.

(i) In order to identify the half plane represented by an inequality, it is just sufficient to take any point (a, b) not on line and check whether it satisfies the inequality or not.
If it satisfies, then the inequality represents the half plane and shade the region which contains the point, otherwise the inequality represents the half plane which does not contain the point within it. For the convenience, the point $(0, 0)$ is preferred.

(ii) If an inequality is of type $ax + by \geq c$ or $ax + by \leq c$, then the point on the line $ax + by = c$ are also included in the solution region. So draw a dark line in the solution region.

(iii) If an inequality is of the type $ax + by > c$ or $ax + by < c$, then the points on the line $ax + by = c$ are not to be included in the solution region. So draw a broken or dotted line in the solution region.

Very Short Answer Type

1. Find a and b such that $2a + i\,4b$ and $2i$ represent the same complex number.
2. Express $(3 + 4i)^{-1}$ in the form $a + ib$.
3. Solve the equation $9x^2 + 16 = 0$ by factorization method.
4. Evaluate i^{-999}.
5. Solve the linear inequation $4x + 3 < 6x + 7$.

Short Answer Type

6. Write $\dfrac{1+7i}{(2-i)^2}$ in the polar form.
7. If $\dfrac{a+ib}{c+id} = x + iy$, then prove that $x^2 + y^2 = \dfrac{a^2+b^2}{c^2+d^2}$.
8. Solve the equation $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}\,i = 0$ using factorisation method.
9. Solve : $\dfrac{3x-4}{2} \geq \dfrac{x+1}{4} - 1$. Also, draw the graph of the solution set.
10. Reduce $\left(\dfrac{1}{1-4i} - \dfrac{2}{1+i}\right)\left(\dfrac{3-4i}{5+i}\right)$ to the standard form.

Long Answer Type

11. (i) Find the complex conjugate of $(2 - 3i)^2$.
(ii) If $a + ib = \dfrac{c+i}{c-i}$, where c is real, prove that $a^2 + b^2 = 1$ and $\dfrac{b}{a} = \dfrac{2c}{c^2-1}$.
12. Evaluate $(-2 + 2\sqrt{-3})^{1/2} + (-2 - 2\sqrt{-3})^{1/2}$
13. (i) If $z_1 = 3 + 4i$ and $z_2 = 12 - 5i$, verify
$$|z_1 + z_2| < |z_1| + |z_2|$$
(ii) Find the conjugate and argument of $\dfrac{2+i}{4i+(1+i)^2}$.
14. Solve the following system of inequations graphically
$3x + 2y \geq 24$, $3x + y \leq 15$, $x \geq 4$
15. (i) Find the real numbers x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.
(ii) Find the modulus and principal argument of the complex number $1 + \cos\alpha + i\sin\alpha$, $\left(0 < \alpha < \dfrac{\pi}{2}\right)$

SOLUTIONS

1. Given, $2a + i\,4b = 0 + i2$
Equating real and imaginary parts, we get
$2a = 0$ and $4b = 2$
$\Rightarrow\ a = 0$ and $b = \dfrac{1}{2}$

2. $(3 + 4i)^{-1} = \dfrac{1}{3+4i} = \dfrac{1}{3+4i} \times \dfrac{3-4i}{3-4i}$
$= \dfrac{3-4i}{9-16i^2} = \dfrac{3-4i}{9+16} = \dfrac{3}{25} - \dfrac{4}{25}i$

3. We have, $9x^2 + 16 = 0 \Rightarrow 9x^2 - 16i^2 = 0$
$\Rightarrow (3x)^2 - (4i)^2 = 0 \Rightarrow (3x + 4i)(3x - 4i) = 0$
$\Rightarrow x = -\dfrac{4}{3}i$, or $x = \dfrac{4}{3}i$

4. $i^{-999} = \dfrac{1}{i^{999}} = \dfrac{1}{(i^2)^{499}i} = \dfrac{1}{(-1)^{499}\cdot i} = \dfrac{1}{-i} = \dfrac{i}{-i^2} = i$

5. The given inequality is $4x + 3 < 6x + 7$.
$\Rightarrow\ 4x - 6x < 7 - 3$
$\Rightarrow\ -2x < 4 \Rightarrow\ x > -2$
$\Rightarrow\ x \in (-2, \infty)$.

6. Let $z = \dfrac{1+7i}{(2-i)^2}$

Now, $z = \dfrac{1+7i}{4+i^2-4i} = \dfrac{1+7i}{3-4i}$

$= \dfrac{(1+7i)(3+4i)}{(3-4i)(3+4i)} = \dfrac{-25+25i}{3^2+4^2} = -1 + i$

$\therefore\ r = |z| = \sqrt{(-1)^2+1^2} = \sqrt{2}$

Again $z = -1 + i$

$\therefore\ x = -1, y = 1.$

Also $\tan\theta = \left|\dfrac{y}{x}\right| = \left|\dfrac{1}{-1}\right| = 1$

$\therefore\ \theta = \dfrac{\pi}{4}$

$\because\ x = -1 < 0$ and $y = 1 > 0$ hence z lies in the 2^{nd} quadrant.

Hence $\arg z = \pi - \theta = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

$\therefore$ Polar form of z is $\sqrt{2}\left(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right)$.

7. We have,

$x + iy = \dfrac{a+ib}{c+id} = \dfrac{a+ib}{c+id} \times \dfrac{c-id}{c-id}$

$= \dfrac{(ac+bd)+i(bc-ad)}{c^2+d^2}$

$= \dfrac{ac+bd}{c^2+d^2} + \dfrac{bc-ad}{c^2+d^2}i$

$\therefore\ x = \dfrac{ac+bd}{c^2+d^2},\ y = \dfrac{bc-ad}{c^2+d^2}$

[Comparing real and imaginary parts]

$\therefore\ x^2 + y^2 = \left(\dfrac{ac+bd}{c^2+d^2}\right)^2 + \left(\dfrac{bc-ad}{c^2+d^2}\right)^2$

$= \dfrac{a^2c^2+b^2d^2+2acbd+b^2c^2+a^2d^2-2bcad}{(c^2+d^2)^2}$

$= \dfrac{(a^2c^2+b^2c^2)+(b^2d^2+a^2d^2)}{(c^2+d^2)^2}$

$= \dfrac{(a^2+b^2)(c^2+d^2)}{(c^2+d^2)^2}$

$\therefore\ x^2 + y^2 = \dfrac{a^2+b^2}{c^2+d^2}$ $\quad [\because\ c^2 + d^2 \neq 0]$

8. We have, $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}\,i = 0$

$\Rightarrow\ (x^2 - 3\sqrt{2}\,x) - 2ix + 6\sqrt{2}\,i = 0$

$\Rightarrow\ x(x - 3\sqrt{2}) - 2i(x - 3\sqrt{2}) = 0$

$\Rightarrow\ (x - 2i)(x - 3\sqrt{2}) = 0$

$\Rightarrow\ x = 2i$ or $x = 3\sqrt{2}$.

Hence, the roots of the given equation are $2i$ and $3\sqrt{2}$.

9. We have, $\dfrac{3x-4}{2} \geq \dfrac{x+1}{4} - 1$

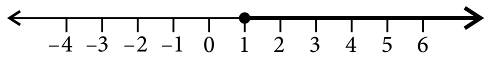
or, $\dfrac{3x-4}{2} \geq \dfrac{x-3}{4}$

or, $2(3x - 4) \geq (x - 3)$

or, $6x - 8 \geq x - 3$

or, $5x \geq 5$ or $x \geq 1$

The graphical representation of solution is given in figure.



10. $\left(\dfrac{1}{1-4i} - \dfrac{2}{1+i}\right)\left(\dfrac{3-4i}{5+i}\right)$

$= \dfrac{[1+i-2(1-4i)]}{(1-4i)(1+i)} \times \left(\dfrac{3-4i}{5+i}\right)$

$= \dfrac{(1-2)+i(1+8)}{1+i(1-4)-4(-1)} \times \dfrac{3-4i}{5+i}$ $\quad (\because\ i^2 = -1)$

$= \dfrac{-1+9i}{5-3i} \times \dfrac{3-4i}{5+i}$

$= \dfrac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}$

$= \dfrac{-3+31i+36}{25-10i+3} = \dfrac{33+31i}{28-10i} \times \dfrac{28+10i}{28+10i}$

$= \dfrac{924+330i+868i+310i^2}{784-100i^2}$

$= \dfrac{924+(330+868)i-310}{784+100}$ $\quad (\because\ i^2 = -1)$

$= \dfrac{614+1198i}{884} = \dfrac{307+599i}{442}$

11. (i) $(2 - 3i)^2 = (2 - 3i)(2 - 3i) = 4 - 6i - 6i + 9i^2$

$= 4 - 12i + 9(-1) = -5 - 12i$

$\therefore$ Complex conjugate of $(2 - 3i)^2$

$= \overline{(-5 - 12i)} = -5 + 12i$

(ii) We have,

$a + ib = \dfrac{c+i}{c-i} = \dfrac{c+i}{c-i} \times \dfrac{c+i}{c+i} = \dfrac{(c+i)^2}{c^2-i^2}$

$= \dfrac{c^2+2ic+i^2}{c^2+1} = \dfrac{c^2-1}{c^2+1} + i\dfrac{2c}{c^2+1}$

Equating real and imaginary parts, we get

$$a = \frac{c^2-1}{c^2+1} \text{ and } b = \frac{2c}{c^2+1}$$

$$\therefore \quad a^2 + b^2 = \left(\frac{c^2-1}{c^2+1}\right)^2 + \frac{4c^2}{(c^2+1)^2}$$

$$= \frac{c^4+1-2c^2+4c^2}{(c^2+1)^2} = \frac{(c^2+1)^2}{(c^2+1)^2} = 1$$

Also, $\frac{b}{a} = \frac{(2c/c^2+1)}{(c^2-1)/(c^2+1)} = \frac{2c}{c^2-1}$

12. Let $a + ib$ be a square root of $(-2 + 2\sqrt{3}\,i)$.

$\therefore \quad (a + bi)^2 = -2 + 2\sqrt{3}\,i$

$\Rightarrow \quad a^2 + 2abi + i^2b^2 = -2 + 2\sqrt{3}\,i$

$\Rightarrow \quad (a^2 - b^2) + 2abi = -2 + 2\sqrt{3}\,i$

$\therefore \quad a^2 - b^2 = -2 \qquad \ldots(i)$

and $\quad 2ab = 2\sqrt{3} \qquad \ldots(ii)$

Now, $\quad (a^2 + b^2)^2 = (a^2 - b^2)^2 + 4a^2b^2$

$= (-2)^2 + (2\sqrt{3})^2 = 4 + 12 = 16$

$\therefore \quad a^2 + b^2 = \sqrt{16} = 4 \qquad \ldots(iii)$

Solving (i) and (iii), we get

$2a^2 = 2$ or $a^2 = 1, \quad \therefore \quad a = \pm 1$

$2b^2 = 6$ or $b^2 = 3 \quad \therefore \quad b = \pm\sqrt{3}$

From (ii), $ab = \sqrt{3}$ which is positive,

$\therefore$ either $a = 1, b = \sqrt{3}$ or $a = -1, b = -\sqrt{3}$.

Hence the two square roots are $1 + \sqrt{3}i$ and $-1 - \sqrt{3}i$.

So $\quad (-2 + 2\sqrt{-3})^{1/2} = \pm(1 + \sqrt{3}\,i) \qquad \ldots(iv)$

$\therefore \quad (-2 - 2\sqrt{-3})^{1/2} = \pm(1 - \sqrt{3}\,i) \qquad \ldots(v)$

Adding (iv) and (v), we get

$(-2 + 2\sqrt{-3})^{1/2} + (-2 - 2\sqrt{-3})^{1/2} = \pm 2$

13. (i) We have, $z_1 = 3 + 4i$ and $z_2 = 12 - 5i$.

$z_1 + z_2 = (3 + 4i) + (12 - 5i) = 15 - i$

$\therefore \quad |z_1 + z_2| = \sqrt{(15)^2+(-1)^2} = \sqrt{225+1} = \sqrt{226}$

Also, $\quad |z_1| + |z_2| = \sqrt{9+16} + \sqrt{144+25}$

$= 5 + 13 = 18$

Hence $|z_1 + z_2| < |z_1| + |z_2|$

(ii) Let $z = \frac{2+i}{4i+(1+i)^2} = \frac{2+i}{4i+1+i^2+2i} = \frac{2+i}{6i}$

$$= \left(\frac{2+i}{6i}\right)\left(\frac{-6i}{-6i}\right) = \frac{6-12i}{36} = \frac{1}{6}-\frac{1}{3}i$$

$\therefore \quad \bar{z} = \frac{1}{6}+\frac{1}{3}i$

Now $\quad z = \frac{1}{6}-\frac{1}{3}i$

$\therefore \quad x = \frac{1}{6}, y = -\frac{1}{3}$

$\tan\theta = \left|\frac{y}{x}\right| = \left|\frac{-1/3}{1/6}\right| = |-2| = 2$

$\therefore \quad \theta = \tan^{-1}2, 0 \le \theta < \frac{\pi}{2}$

$\because \quad x = \frac{1}{6} > 0$ and $y = -\frac{1}{3} < 0$. Hence z lies in the 4th quadrant.

$\therefore \quad \arg z = 2\pi - \theta = 2\pi - \tan^{-1}2.$

14. Given inequations are

$3x + 2y \ge 24 \qquad \ldots(i)$

$3x + y \le 15 \qquad \ldots(ii)$

and $\quad x \ge 4 \qquad \ldots(iii)$

Corresponding equations are

$l_1 : 3x + 2y = 24 \qquad \ldots(iv)$

$l_2 : 3x + y = 15 \qquad \ldots(v)$

$l_3 : x = 4 \qquad \ldots(vi)$

Line (iv) cuts x-axis at $A(8, 0)$ and y-axis at $B(0, 12)$.

Line (v) cuts x-axis at $C(5, 0)$ and y-axis at $D(0, 15)$.

Line (vi) is parallel to y-axis cutting x-axis at $E(4, 0)$.

Point $O(0, 0)$ lies on none of the lines (iv), (v) and (vi).

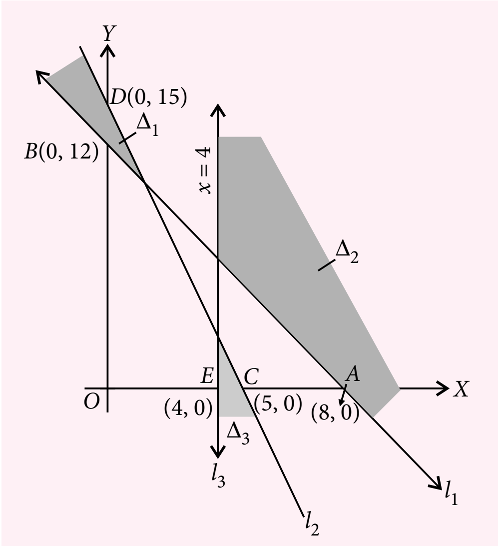
For $O(0, 0)$, inequation $3x + 2y \ge 24$ becomes $3\cdot0 + 2\cdot0 \ge 24$, or $0 \ge 24$, which is not true.

Therefore, inequation (i) represents that half-plane made by the line (iv) which does not contain the origin.

For $O(0, 0)$, inequation $3x + y \le 15$ becomes $3\cdot0 + 0 \le 15$ or $0 \le 15$, which is true.

Therefore, inequation (ii) represents that half-plane made by the line (v) which contains the origin.

Inequation $x \ge 4$ represents the half-plane on the right side of the line $x = 4$ (which is parallel to y-axis).



Δ_1 is the region common to the half-planes representing inequations (i) and (ii).
Δ_2 is the region common to the half-planes representing inequations (i) and (iii).
Δ_3 is the region common to the half-planes representing inequations (ii) and (iii).
Clearly no point or region is common in regions $\Delta_1, \Delta_2, \Delta_3$.
Hence solution set of given system of inequations $= \phi$.

Thus no solution of the given system of inequations exists.

15. (i) Conjugate of $-6 - 24i$ is $-6 + 24i$ ...(i)

Also, $(x - iy)(3 + 5i) = 3x - 5yi^2 - 3yi + 5xi$

$= (3x + 5y) + (5x - 3y)i$ ...(ii)

From (i) and (ii), we have

$3x + 5y + (5x - 3y)i = -6 + 24i$

$\therefore$ $3x + 5y = -6$ and $5x - 3y = 24$

or $3x + 5y + 6 = 0$ and $5x - 3y - 24 = 0$

$$\Rightarrow \frac{x}{-120+18} = \frac{y}{30+72} = \frac{1}{-9-25}$$

$$\text{or } \frac{x}{-102} = \frac{y}{102} = \frac{1}{-34}$$

$$\therefore x = \frac{-102}{-34} = 3,\ y = \frac{102}{-34} = -3$$

$\therefore$ $x = 3, y = -3$

(ii) $$1 + \cos\alpha + i\sin\alpha = 2\cos^2\frac{\alpha}{2} + i2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

$$= 2\cos\frac{\alpha}{2}\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right)$$

$$= r(\cos\theta + i\sin\theta),$$

where $r = 2\cos\frac{\alpha}{2}$, $\theta = \frac{\alpha}{2}$.

Since $-\pi < \frac{\alpha}{2} \le \pi$, so $\frac{\alpha}{2}$ is the principal argument of given complex number and $2\cos\frac{\alpha}{2}$ is the modulus.