Sets, Relations and Functions

Sets

Empty set $E = \{\ \} = \phi; n(E) = 0$

Singleton set $S = \{4\}; n(S) = 1$

Finite set $F = \{1, 2, 3, 4, 5\}; n(F) = 5$

Infinite set $N = \{1, 2, 3, \ldots\}; n(N) = infinity$

Equal set *Two sets having same elements; set of letters in the word heart and earth*

$E = \{e, a, r, t, h\}$ $H = \{h, e, a, r, t,\}$ $H = E$

Equivalent set *Two sets having same number of elements, but different elements;*

~~*set of letters in the word heart and earth*~~

$E = \{e, a, r, t, h\}$ $F = \{1, 2, 3, 4, 5\}$ $F \approx E$

Sets, Relations and Functions

Sets

Universal set — *Contains all the elements of which all other sets are subsets*

$A\ set\ of\ all\ vowels, U = \{a, e, i, o, u\}$

$A = \{a, e, i\}; B = \{i, o, u\}$

Subset set — *A set A is a subset of B only if every element in A is also an element in B*

$A \subseteq B \quad B = \{s, i, r\};$

$Subset\ sets:\ 2^n\ subsets:$

$A_1 = \{s, i, r\}; \quad A_2 = \{s, i\}; \quad A_3 = \{s, r\}; \quad A_4 = \{i, r\};$

$A_5 = \{s\}; \quad A_6 = \{r\}; \quad A_7 = \{i\}; \quad A_8 = \{\ \};$

Proper subset — *if atleast one element in B is not contained in A*

$A = \{1, 3, 5\}; B = \{1, 3, 5, 7\}; \quad A \subset B$

$G = \{h, a, b\}; H = \{h, b\}; \quad H \subset G \quad n(Proper\ subset) = 2^n - 1$

Sets, Relations and Functions

Sets

Super set — *A set containing all the elements of another set.*

$A \supseteq B \quad B = \{s, i, r\}; A = \{s, i, r, d\}$

Proper superset — $A_5 = \{s\}; \quad A_6 = \{r\}; \quad A_7 = \{i\}; \quad A_8 = \{\ \};$

if atleast one element in B is not contained in A

$A = \{1, 3, 5, 7\}; B = \{1, 3, 5\}; \quad A \supseteq B \text{ and } A \neq B \rightarrow A \supset B$

Power set — *Power set contains all the subsets of a set* $n(P) = 2^n$

Sets, Relations and Functions

Sets

Cartesian Product of sets

Relations

Functions

Relations

Definition Types Domain Range

Functions

Definition Types Domain Range

Sets, Relations and Functions

Cartesian Product of two sets

$A = \{1,2\}$

B= $\{5,7,9\}$

Set $\rightarrow A \times B = \{(1,5),(1,7),(1,9),(2,5),(2,7),(2,9)\}$

Since each one pair is a single element, use curly bracket.

It will come in pairs, one from first set, second from second set

This is also a set.

Sets, Relations and Functions

Cartesian Product of two sets

Example 1:

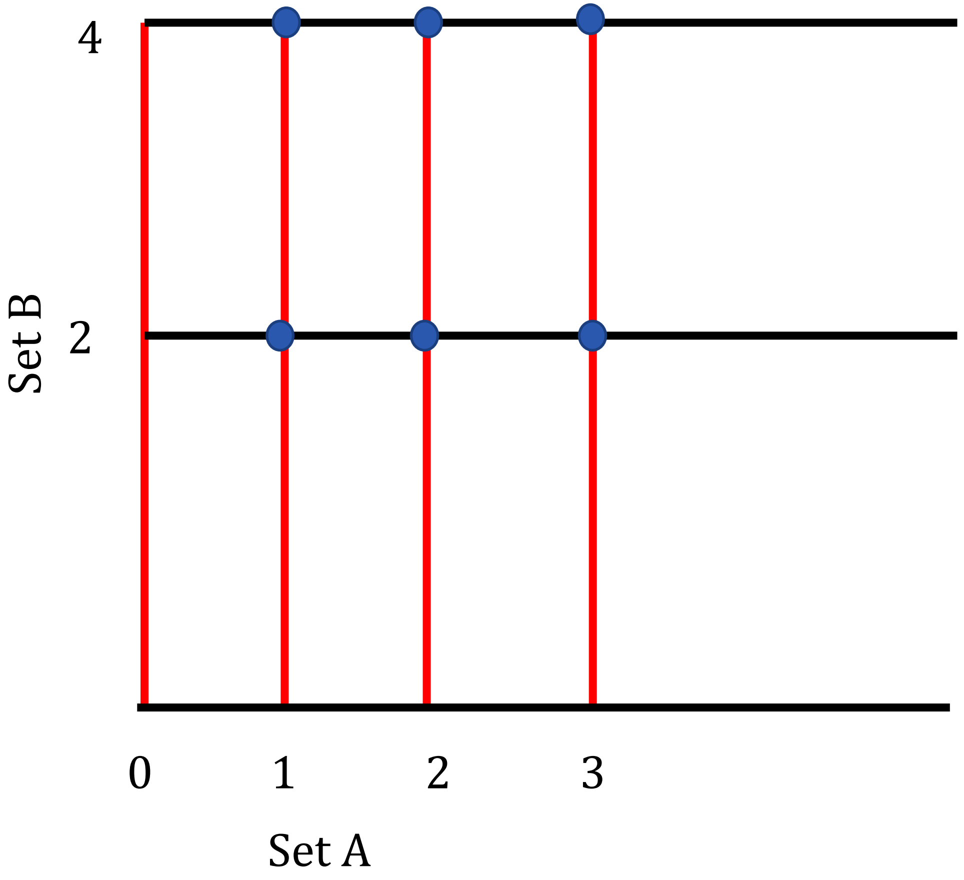
If $A = \{1,2,3\}$ and $B = \{2,4\}$ then the *Cartesian product* $A \times B$ *is*

A $\{(1, 2),(1, 4),(2, 2), (2, 4), (3, 2), (3, 4)\}$

B $\{(2,1),(4,1),(2, 2), (2, 4), (3, 2), (3, 4)\}$

C $\{(1, 2),(1, 4),(2, 2), (2, 4), (3, 2), (4,3)\}$

D $\{(1, 2),(1, 4),(2, 2), (2, 4), (3, 2), (3, 4)\}$



A $\{(1, 2),(1, 4),(2, 2), (2, 4), (3, 2), (3, 4)\}$

Sets, Relations and Functions

Cartesian Product of two sets

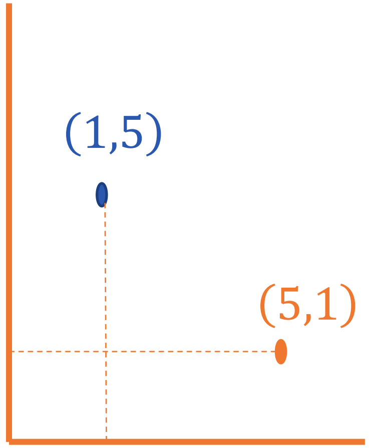
$A = \{1,2\}$

$B = \{5,7,9\}$

Set $\rightarrow A \times B = \{(1,5), (1,7), (1,9), (2,5), (2,7), (2,9)\}$

Set $\rightarrow B \times A = \{(5,1), (5,2), (7,1), (7,2), (9,1), (9,2)\}$

$A \times B \neq B \times A$



Sets, Relations and Functions

Cartesian Product of two sets

Donot make mistake in writing the pair properly.

How to avoid silly mistake? Take it seriously!, You won't make it again

Sets, Relations and Functions

Cartesian Product of two sets

Empty set : $\{.\}$ No element is there in the empty set

Consider two non – empty sets A and B $A = \{a_1, a_2, a_3\}$ $B = \{b_1, b_2, b_3\}$

Carttesian product of sets A and B = $A \times B = \{(a, b): a \in A \text{ and } b \in B\}$

Example 2a:

if $A = \{2,3\}$ and $B = \{a, b, c\}$, find $A \times B$

$A \times B = \{(2, a), (2, b), (2, c), (3, a), (3, b), (3, c)\}$

Example 2b:

if $A = \{2,3\}$, find $A \times A$ $A = \{2,3\}$ $A = \{2,3\}$

$A \times A = \{(2,2), (2,3), (3,2), (3,3)\}$

Sets, Relations and Functions

Cartesian Product of two sets

Number of elements in a set: $n(A)$

If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = p(q)$

Example 3:

if $A = \{2,3\}$ and $B = \{5,7,9\}$, find $n(A \times B)$

$\mathrm{A} \times B = \{(2,5), (2,7), (2,9), (3,5), (3,7), (3,9)\}$

$n(A) = 2$ and $n(B) = 3$, hence $n(A \times B) =$ 2x3=6

$n(A) = 2$ and $n(B) = 3$, hence $n(A \times B) = 6$

With 2, 3 times pair, with 3 again 3 pair. So totally 6 pair.

Sets, Relations and Functions

Cartesian Product of three sets

Can you do triplet logically? Yes.

$n(A \times B \times C) = n(A)\, n(B)\, n(C)$

Example 4:

if $A = \{2,3\}, B = \{3,5\}$ and $C = \{7\}$, find $n(A \times B \times C)$

$\mathrm{A} \times B \times C = \{(2,3,7), (2,5,7), (3,3,7), (3,5,7)\}$

$n(A) = 2\,, n(B) = 2$ and $n(C) = 1$ $hence$ $n(A \times B \times C) = 2(2)(1) = 4$

Sets, Relations and Functions

Subsets

Number of subsets of $A = 2^{n(A)}$

Example 5:

If $A = \{2,3\}$ and $B = \{3,5,7\}$ then the number of subsets of $A \times B$

A) 6 B) 2^5 C) 2^6 D) 2^3

$elements\ in\ (A \times B)$:

$\{(2,3),(2,5),(2,7),(3,3),(3,5),(3,7)\}$

$n(A) = 2; n(B) = 3$

$\therefore n(A \times B) = 2(3) = 6$

Number of subsets of $A \times B = 2^{n(A \times B)} = 2^6$

Sets, Relations and Functions

Relations

$A = \{1,2\}$ $B = \{3,4,5\}$

$A \times B = \{(1,3),(1,4),(1,5),(2,3),(2,4),(2,5)\}$

Create any random subset R_1 from $A \times B$

$R_1 = \{(1,3),(1,5),(2,5)\}$

$A \times B$ has six elements. Hence it has 2^6 subsets.

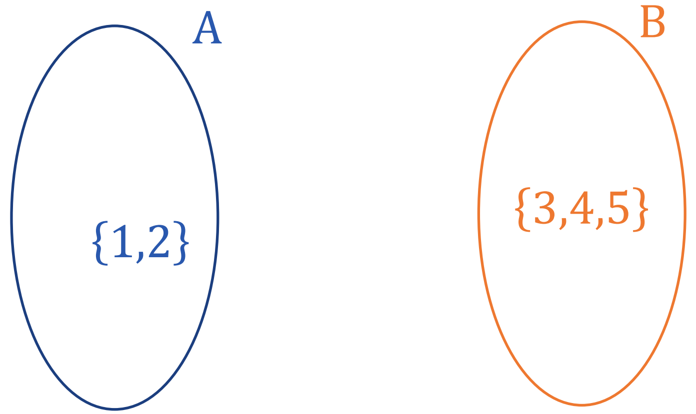
All these subsets are relations

Why do you name it as relation?

Sets, Relations and Functions

Relations

Let us show in a Venn diagram



Definition:

Let A and B be two non − empty subsets.

Any subset of $A \times B$ is called relation from A to B.

Sets, Relations and Functions

Relations

Let $A = \{1,3,5\}$ and $B = \{7,8\}$, then

$A \times B = \{(1,7), (1,8), (3,7), (3,8), (5,7), (5,8)\}$

Consider any random subset,

$R_1 = \{(1,7), (3,7), (3,8)\}$

$R_2 = \{(1,7)\}$

R_1, R_2 being subsets of $A \times B$. are relations from A *to* B.

Sets, Relations and Functions

Relations

Number of subsets of $A = 2^{n(A)}$

Number of relations from $A \; to \; B = 2^{n(A)\,n(B)}$

Example 6:

If $A = \{2,3\}$ and $B = \{3,5,7\}$ then the number of relations from $A \; to \; B$

A) 6 B) 2^5 C) 2^6 D) 2^3

Number of elements of $A \times B = n(A)\,n(B) = 2(3) = 6$

Number of relations from $A \; to \; B =$ Number of subsets of $A \times B = 2^6$

Sets, Relations and Functions

Relations

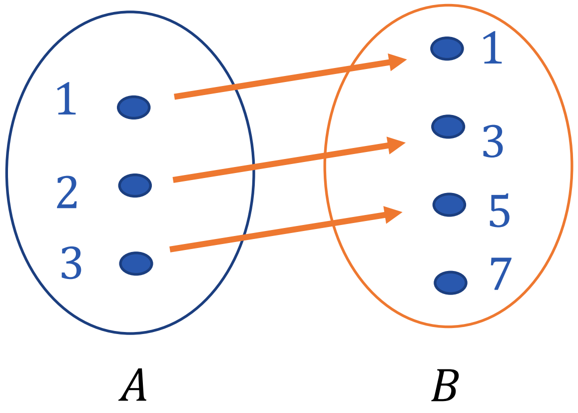
Description of Relations

There are three ways to describe a relation.

(a) $R_1 = \{(1,1), (2,3), (3,5), \}$

(b) $R_2 = \{(x, y) : y = 2x - 1, x \in A \text{ and } y \in B\}$

(c) $R_3 = \{xRy \Leftrightarrow y = 2x - 1, x \in A \text{ and } y \in B\}$



Sets, Relations and Functions

Relations

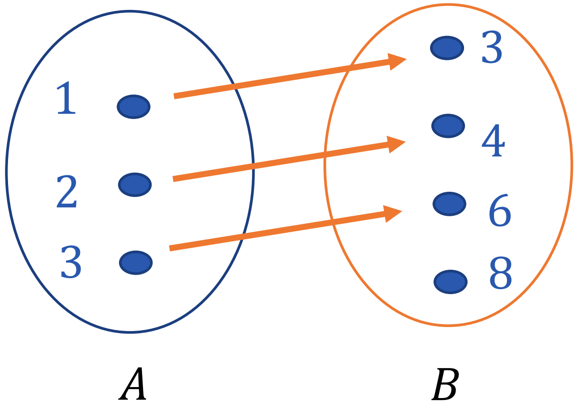
(a) If $(x, y) \in R$, then we call y as the image of x and x as the pre − image of y

(b) Relation from A to A ($i.e.$ subset of $A \times A$) is also called 'relation on A'.

What is the image of 3? What is the preimage of 3?

What is 3 to 6?

preimage or image?



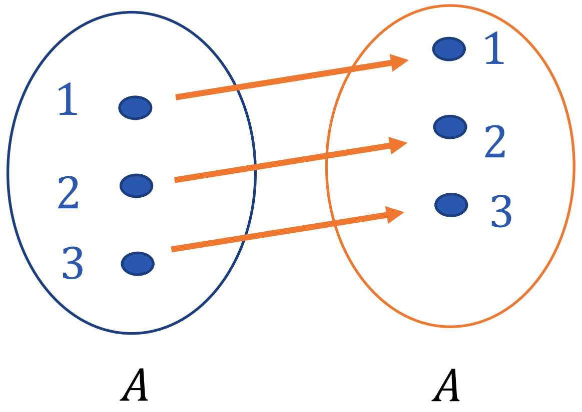
Sets, Relations and Functions

Relations

Subset of $A \times B$ is relation from $A \to B$

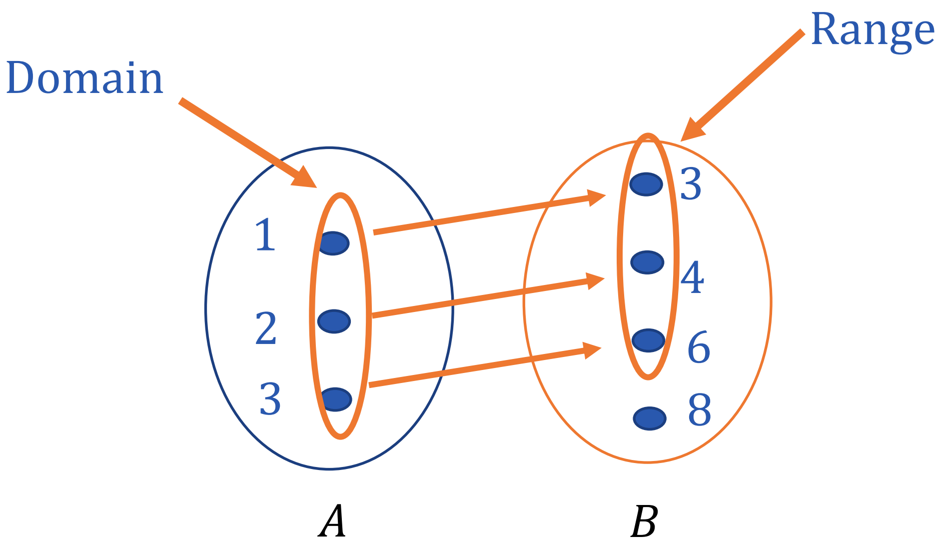
Subset of $A \times A$ is relation from $A \to A$

Subset of $A \times A$ is also called relation on A



Sets, Relations and Functions

Domain and Range of a Relation



Sets, Relations and Functions

Domain and Range of a Relation

$A = \{1,2,3,4\}$

$B = \{5,6,8,9\}$

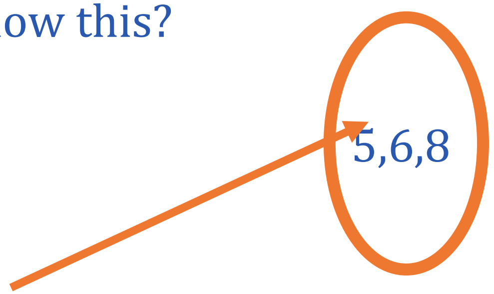
$R = \{(1,5), (1,6), (2,8)\}$

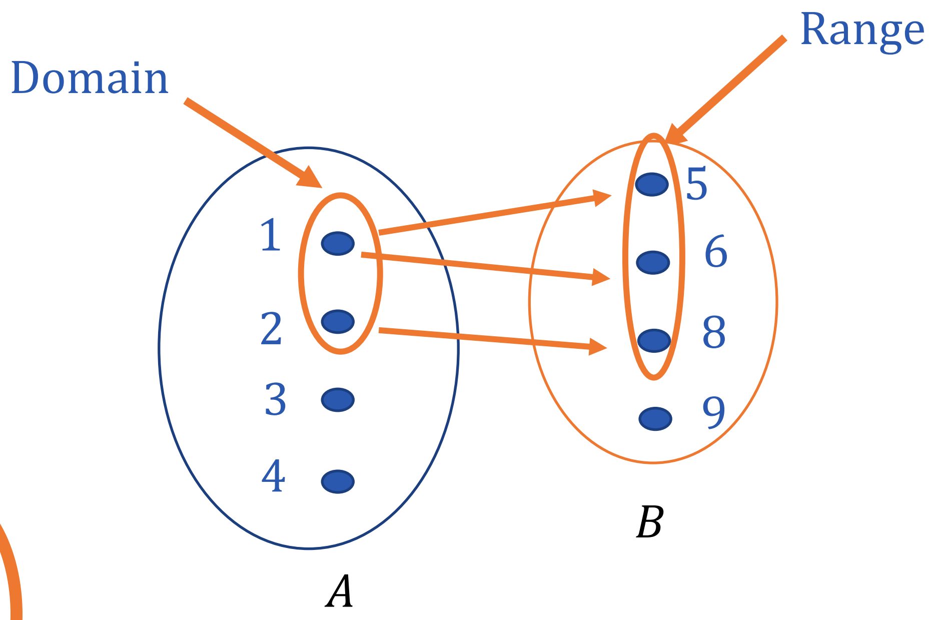
$D_R = \{1,2\}$ $R_R = \{5,6,8\}$

How do you show this?

Domain ? {1,2}

Range ? {5,6,8}





Sets, Relations and Functions

Domain and Range of a Relation

Domain of a $R = \{x : (x, y) \in R\}$

Domain of a $R = \{x : (x, y) \in \mathbb{R}\}$

If R is a relation from $A \ to \ B$, then B is called $co - domain$.

Sets, Relations and Functions

Domain and Range of a Relation

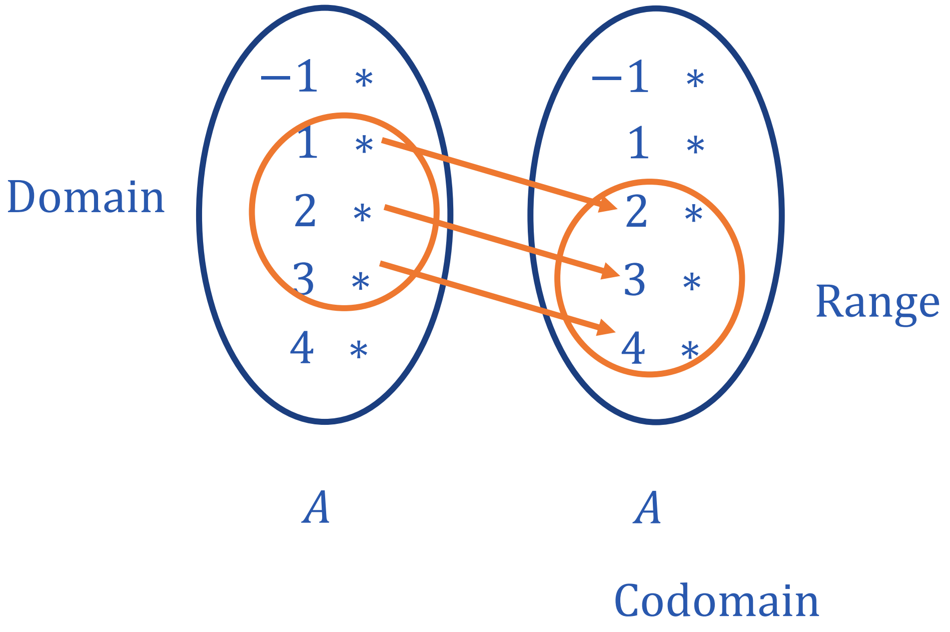
$$Consider\ a\ relation\ x\ R\ y \iff y = x + 1\ defined\ on\ A = \{-1,1,2,3,4\}$$

x is in relation to y

$$x\ R\ y = \{(1,2),(2,3),(3,4)\}$$

Domain of $R = \{1,2,3\}$

Range of $R = \{2,3,4\}$



Sets, Relations and Functions

Domain and Range of a Relation

Given $R = \{(x, y) \in W, x^2 + y^2 = 25\}$. Find the domain and range of R. $W = \{0,1,2,3,4,5 \ldots\}$

$x = 0 \rightarrow y = 5$ √

$x = 1 \rightarrow y = \sqrt{24} \neq W$ ×

$x = 2 \rightarrow y = \sqrt{21} \neq W$ ×

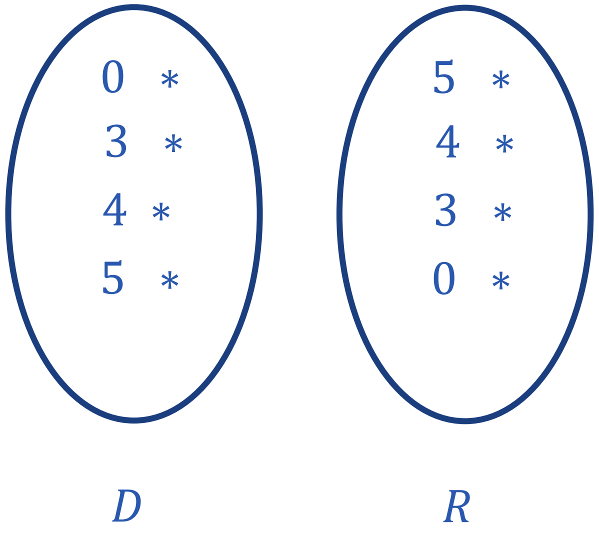
$x = 3 \rightarrow y = 4$ √

$x = 4 \rightarrow y = 3$ √

$x = 5 \rightarrow y = 0$ √

Domain = $\{0,3,4,5\}$

Range = $\{5,4,3,0\}$



$R = \{(0,5), (3,4), (4,3)(5,0)\}$

Sets, Relations and Functions

Types of Relation

Subsets of $A \times B$

Subsets of $A \times A$

Do you understand this?

Empty relation

Universal relation

Identity relation

Sets, Relations and Functions

Types of Relation

Reflexive Relation

Symmetric Relation

Transitive Relation

Equivalence Relation

Sets, Relations and Functions

Types of Relation

Here we talk about relation on A only

(a) Empty Relation

Let A be a set and $\emptyset \subset A \times A$. Then, $\emptyset$ is a relation on A.

$\emptyset$ is called empty relation.

$\emptyset = \{.\}$

(b) Universal Relation

Let A be a set and $A \times A \subseteq A \times A$. Then, $A \times A$ is a relation on A.

$A \times A$ is called universal relation.

Let $A = \{1,2\}$

$R_2 = A \times A = \{(1,1),(1,2),(2,1),(2,2)\}$

(c) Identity Relation

Let A be a set. If every element of A is related to itself only, then it is called identity relation.

$I_A = \{(a,a): a \in A\}$ is called identity relation on A

$R_3 = \{(1,1),(2,2)\}$

Sets, Relations and Functions

Types of Relation

Let $A = \{2,3\}$

$A \times A = \{(2,2), (2,3), (3,2), (3,3)\}$

$R_1 = \emptyset = \{.\}$ called empty relation

$R_2 = A \times A = \{(2,2),(2,3),(3,2),(3,3)\}$ called universal relation

$R_3 = \{(2,2), (3,3)\}$ called identity relation

$R_3 = \{(3,3)\}$ is not identity relation

Sets, Relations and Functions

Functions

Functions

Definition Types Domain Range

You already know what is a function?

$f(x) = x^2$ This is a function!

$f(2) = 2^2 = 4$

$f(3) = 3^2 = 9$

$f(10) = 10^2 = 100$

$f(x) = 2x + 1$ This is a function!

$f(2) = 2(2) + 1 = 5$

$f(3) = 2(3) + 1 = 7$

$f(10) = 2(10) + 1 = 21$

Now formally, we will see the terms and study

Sets, Relations and Functions

Functions

Function is a relation having two particular characteristics.

Function is a relation only, but under two conditions

Sets, Relations and Functions

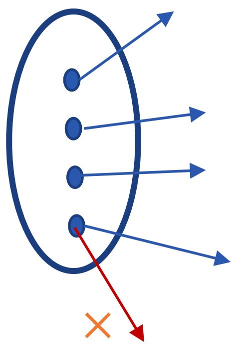
Function

Let A and B be two nonempty sets.

A function from A to B, i.e., $\mathrm{f}: \mathrm{A} \rightarrow B$ is a relation such that

(a) all the elements of A are related to the elements of B and

(b) no element of A is related to more than one element of B.



Sets, Relations and Functions

Function

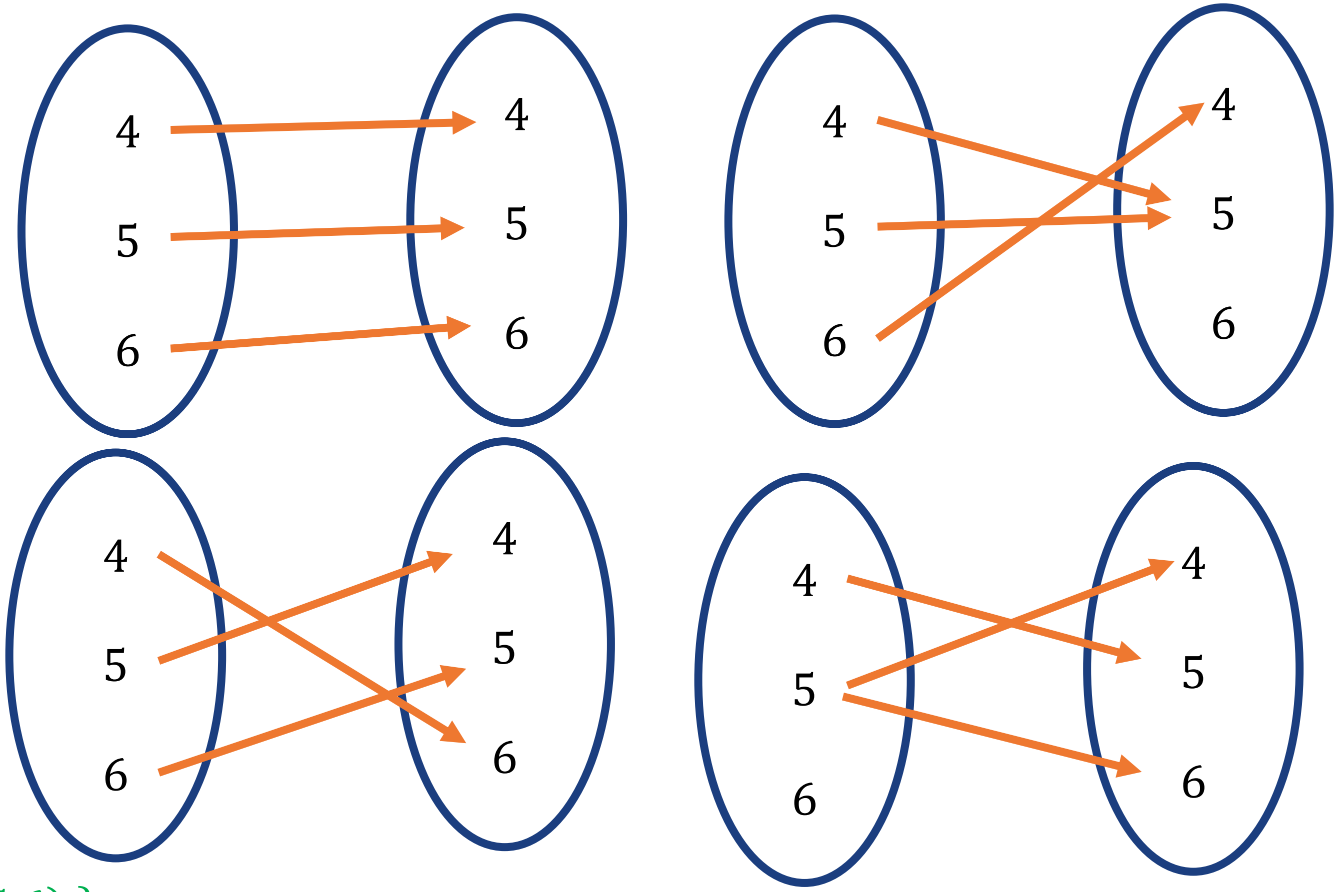
Which of the following relations on the set {4,5,6} is not a function on the same set? Why?

A {(4,4), (5,5), (6,6) }

B {(4,5), (5,5), (6,4) }

C {(4,6), (5,4), (6,5) }

D {(4,5), (5,4), (5,6) }



D {(4,5), (5,4), (5,6) }Two reasons! 5 is friend with two; 6 is alone

Sets, Relations and Functions

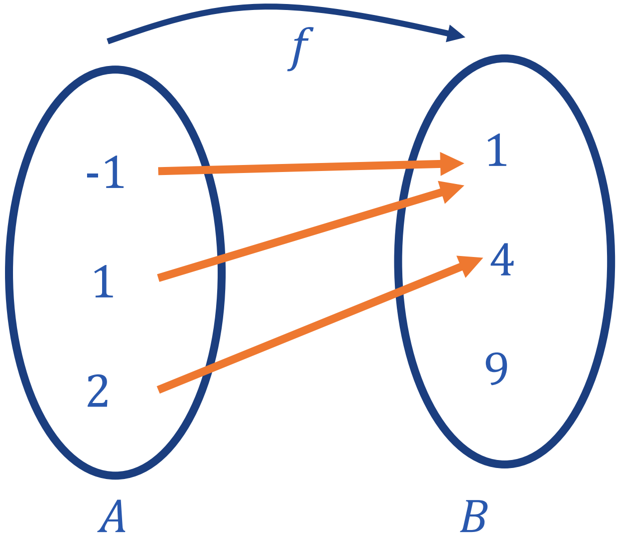
Function

$f: A \to B$, where $A = \{-1,1,2\}$ and $B = \{1,4,9\}$ defined by $f(x) = x^2$ is a function for which

(a) $A = \{-1,1,2\}$ is called the domain of f

(b) $B = \{1,4,9\}$ is called the codomain of f

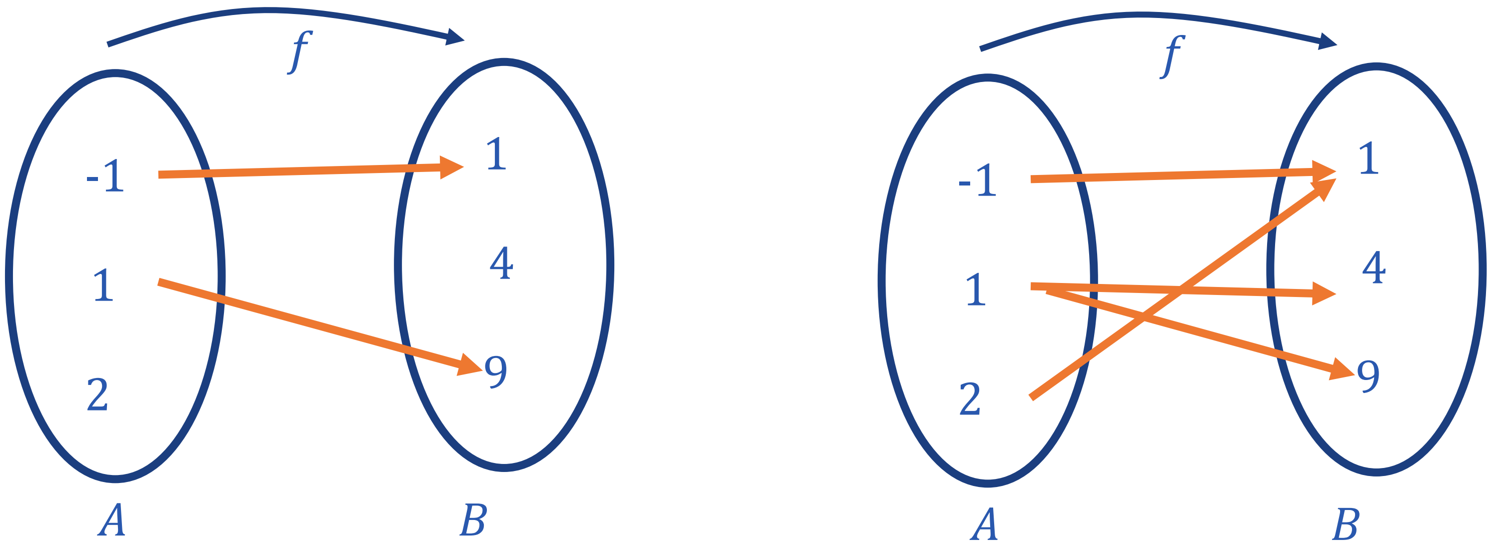
(c) $\{1,4\}$ is called the range of f, which is a collection of all images.



B *is codmain;*

Sets, Relations and Functions

Function



2 has no image

1 has more than one image

Both are relations, but neither is a function from A to B.

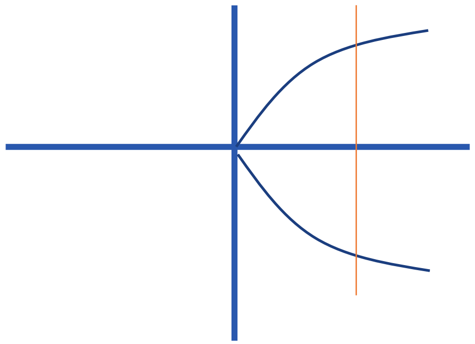
Sets, Relations and Functions

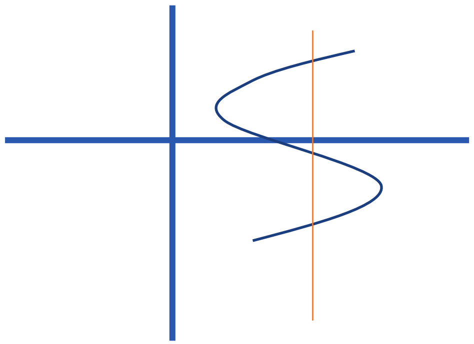
Function

Neither of the graphs given here represent functions.

Vertical line test. If a vertical line cuts the graph atmost one point, it is a function.

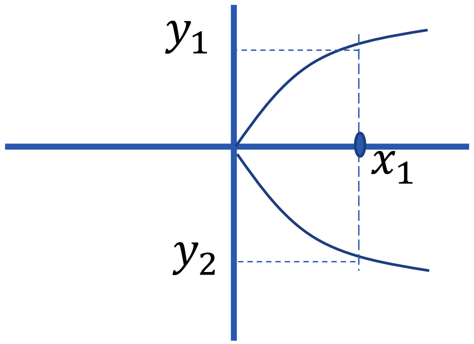
In a function, every input from the domain has unique output from the codomain.



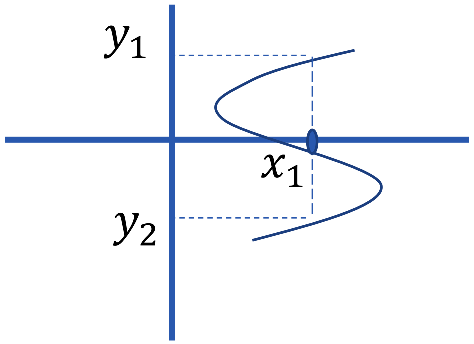


Sets, Relations and Functions

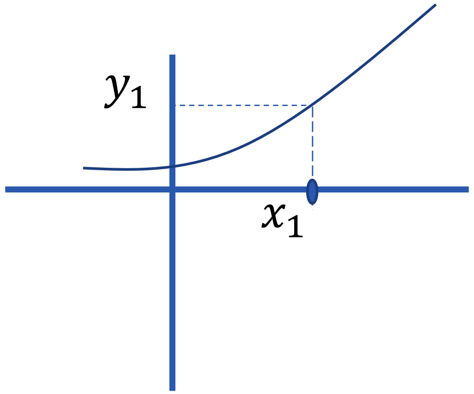
Function



$f(x_1) = y_1, y_2,$



$f(x_1) = y_1, y_2,$



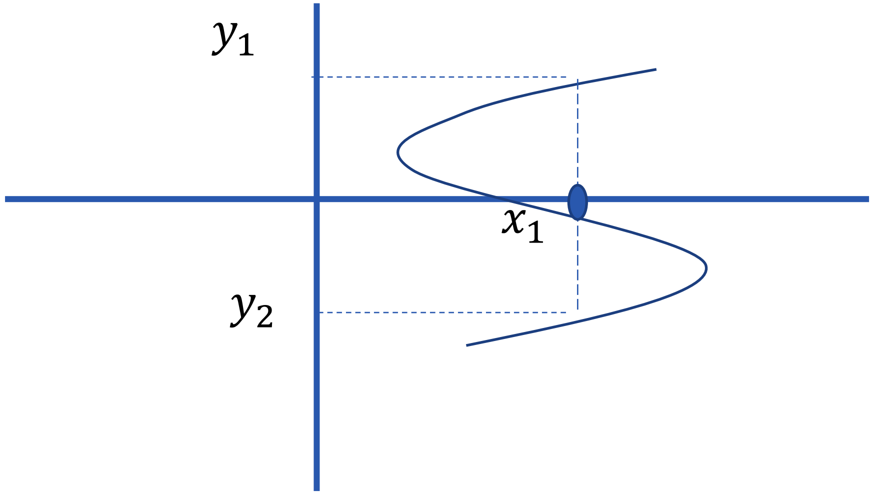
$f(x_1) = y_1$

Vertical Line Test

Sets, Relations and Functions

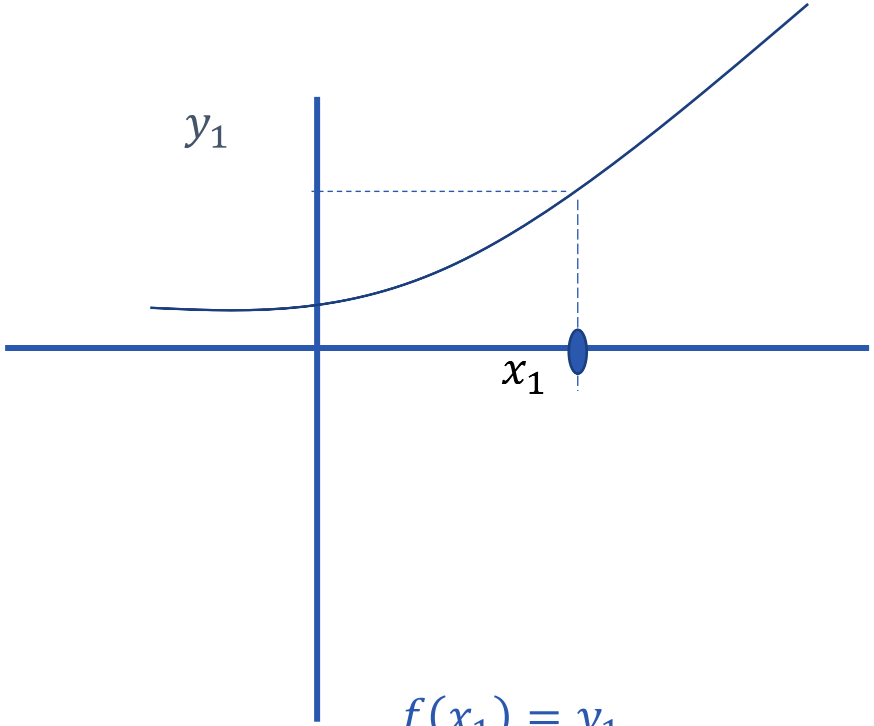
Graph of Functions

Is this grpah of a function?



$f(x_1) = y_1, y_2,$

Is this grpah of a function?



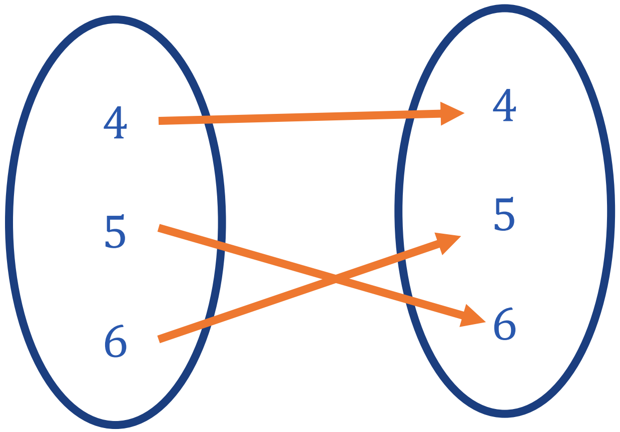
$f(x_1) = y_1$

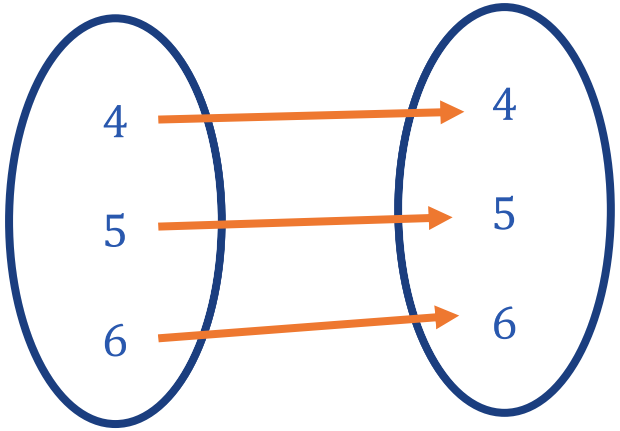
Sets, Relations and Functions

Classification of Functions

Quiz:

Which one is true?





one to many ? is not a function at all

onto (every one is covered)

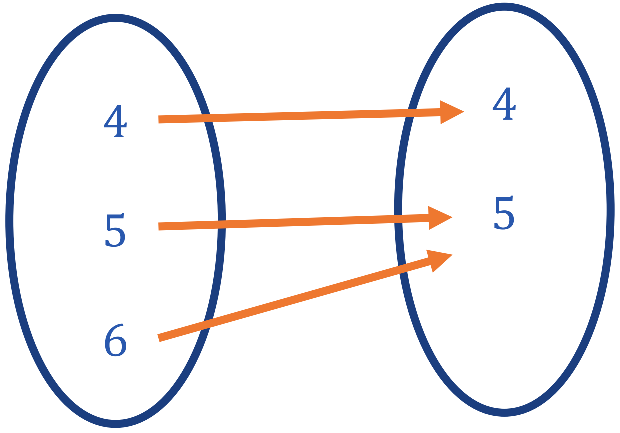
into (some remains uncovered in codomain)

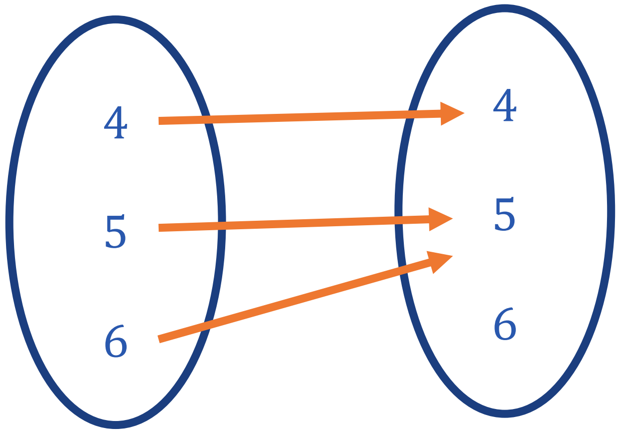
onto (everyone is covered)

Sets, Relations and Functions

Classification of Functions

Quiz:





many to one and onto

many to one and into

onto (every one is covered)

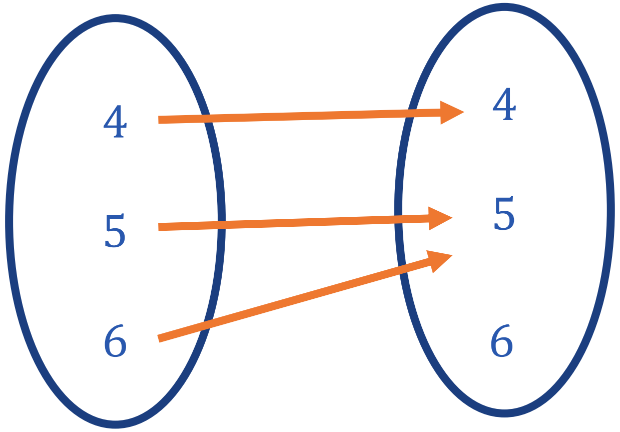
into (some remains uncovered)

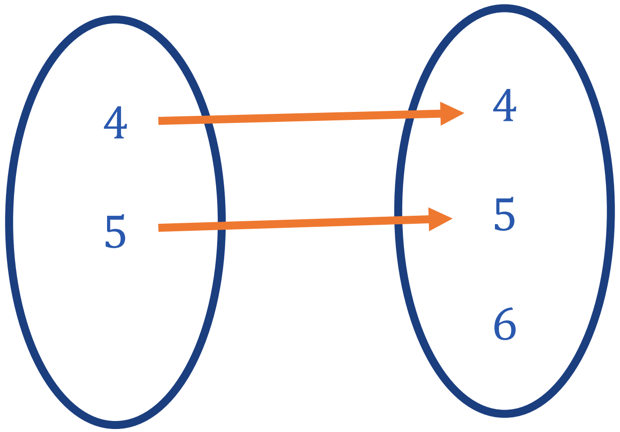
one to many ? is not a function at all

Sets, Relations and Functions

Classification of Functions

Quiz:





many to one and into

one to one and into

into (some remains uncovered)

one to many ? is not a function at all

A. ALGEBRAIC FUNCTION

Function	Domain	Range
(i) x^n, $(n \in N)$	R = set of real numbers	R, if n is odd $R^+ \cup \{0\}$, if n is even
(ii) $\frac{1}{x^n}$, $(n \in N)$	$R - \{0\}$	$R - \{0\}$, if n is odd R^+, if n is even
(iii) $x^{1/n}$ $(n \in N)$	R, if n is odd $R^+ \cup \{0\}$, if n is even	R, if n is odd $R^+ \cup \{0\}$, if n is even
(iv) $\frac{1}{x^{1/n}}$, $(n \in N)$	$R - \{0\}$, if n is odd R^+, if n is even	$R - \{0\}$, if n is odd R^+, if n is even

B. TRIGONOMETRIC FUNCTION

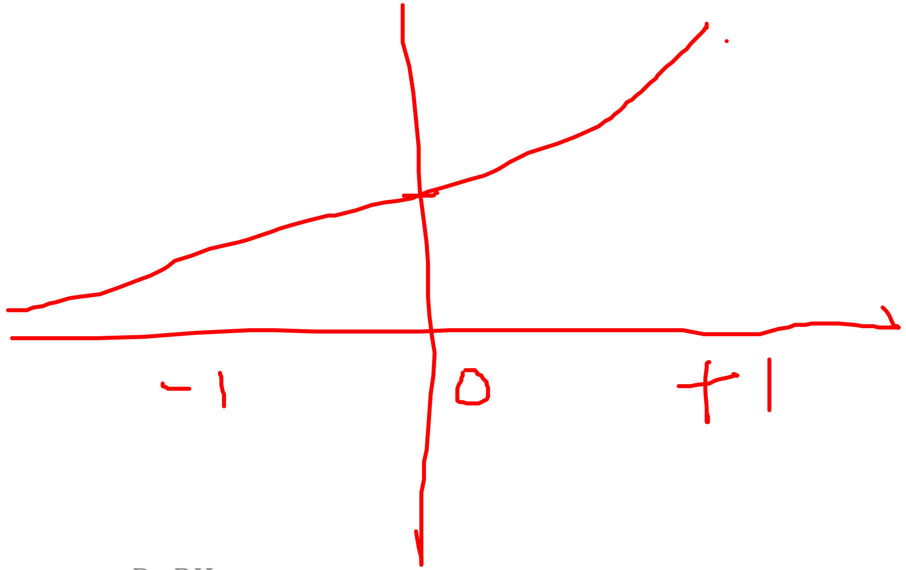
Function	Domain	Range
(i) sin x	R	[–1, 1]
(ii) cos x	R	[–1, 1]
(iii) tan x	$R - (2k+1)\frac{\pi}{2}, k \in I$	R
(iv) sec x	$R - (2k+1)\frac{\pi}{2}, k \in I$	$(-\infty, -1] \cup [1, \infty)$
(v) cosec x	$R - k\pi, , k \in I$	$(-\infty, -1] \cup [1, \infty)$
(vi) cot x	$R - k\pi, , k \in I$	R

C. INVERSE TRIGONOMETRIC FUNCTION

Function	Domain	Range
(i) $\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$
(ii) $\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
(iii) $\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(iv) $\sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left[\frac{\pi}{2}\right]$
(v) $\text{cosec}^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
(vi) $\cot^{-1} x$	R	$(0, \pi)$

D. EXPONENTIAL FUNCTION

Function	Domain	Range
(i) e^x	R	R^+
(ii) $e^{1/x}$	R – {0}	R^+ – {1}
(iii) a^x, a > 0	R	R^+
(iv) $a^{1/x}$, a > 0	R – {0}	R^+ – {1}



E. LOGARITHIMIC FUNCTION

Function	Domain	Range
(i) $\log_a x$, $(a > 0)(a \neq 1)$	R^+	R
(ii) $\log_x a = \dfrac{1}{\log_a x}$ $(a > 0)\ (a \neq 1)$	$R^+ - \{1\}$	$R - \{0\}$

R^+ : Here zero and all $R < 0$ not included

F. INTEGRAL PART FUNCTION

Function	Domain	Range
(i) $[x]$	R	I
(ii) $\frac{1}{[x]}$	$R - [0, 1)$	$\left\{\frac{1}{n}, n \in I - \{0\}\right\}$

$[x]$ *is GIF*, $\{x\}$ *is FPF*

G. FRACTIONAL FUNCTION

Function	Domain	Range
(i) $\{x\}$	R	$[0, 1)$
(ii) $\frac{1}{\{x\}}$	R – I	$(1, \infty)$

H. MODULUS FUNCTION

Function	Domain	Range
(i) $\lvert x \rvert$	R	$R^+ \cup \{0\}$
(ii) $\frac{1}{\lvert x \rvert}$	$R - \{0\}$	R^+

I. SIGNUM FUNCTION

Function	Domain	Range
$\text{sgn}(x) = \frac{\lvert x \rvert}{x}$	R	{–1, 0, 1}

J. CONSTANT FUNCTION

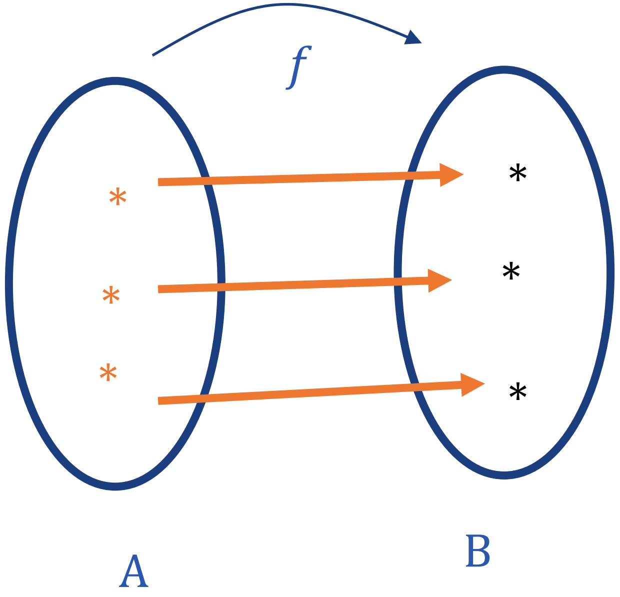
Function	Domain	Range
$f(x) = c$	R	{c}

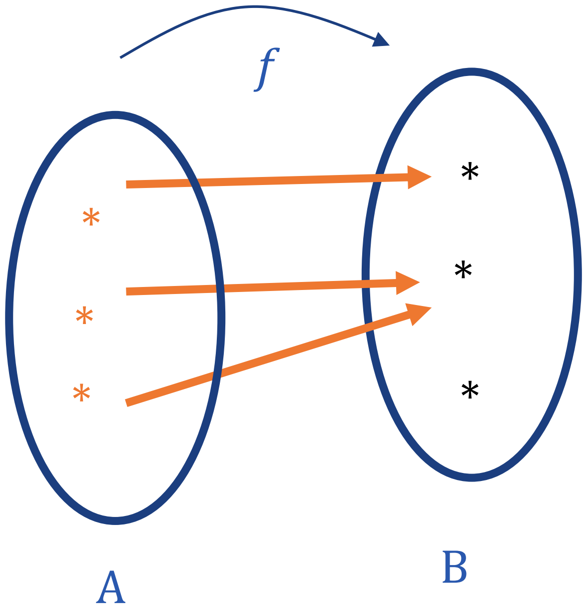
Sets, Relations and Functions

Classification of Functions

Injectivity or one to one

If no two inputs have the same image, then the function is called one – one (or injective), otherwise, it is many – one.





Sets, Relations and Functions

Classification of Functions

Example 6:

First understand, then we will see how it is used!

one − one functions $\rightarrow y = x^3$, y = 2x − 1,..

$$y(+2) = 8; y(-2) = -8$$

many − one functions $\rightarrow y = x^2$, y = cos x, sin x, . . .

$$y(+2) = 4; y(-2) = 4$$

To check the function, first solve

$$f(x_1) = f(x_2)$$

If you get $x_1 = x_2$ only, then it is one − one function, otherwise it is many to one.

Sets, Relations and Functions

Classification of Functions

Check whether the following functions are one – one or many – one.

(a) $f(x) = x^2$

(b) $f(x) = x^5$

(c) $f(x) = x^2, x \in R^+$

Sets, Relations and Functions

Classification of Functions

$Check\ whether\ the\ following\ functions\ are\ one - one\ or\ many - one.$

$(a)\ f(x) = x^2$

$f(x_1) = f(x_2)$

${x_1}^2 = {x_2}^2$

${x_1}^2 - {x_2}^2 = 0$

$(x_1 + x_2)(x_1 - x_2) = 0$

$x_1 = x_2\ or\ x_1 = -x_2$

many – one

Two different x values love the same y!

$(b)\ f(x) = x^5$

$f(x_1) = f(x_2)$

${x_1}^5 = {x_2}^5$

$x_1 = x_2$

one – one

how it is one to one?

Later we will learn this

$(c)\ f(x) = x^2, x \in \mathbb{R}^+$

$f(x_1) = f(x_2)$

${x_1}^2 = {x_2}^2$

${x_1}^2 - {x_2}^2 = 0$

$(x_1 + x_2)(x_1 - x_2) = 0$

$x_1 = x_2\ or\ x_1 = -x_2$

$-x_2 \notin \mathbb{R}^+$

$x_1 = x_2$

one – one

Sets, Relations and Functions

Classification of Functions

$f(x_1) = f(x_2)$

$\downarrow$

$x_1 = x_2 \, or x_1 = -x_2$

many to one

$f(x_1) = f(x_2)$

$\downarrow$

$x_1 = x_2$

one to one

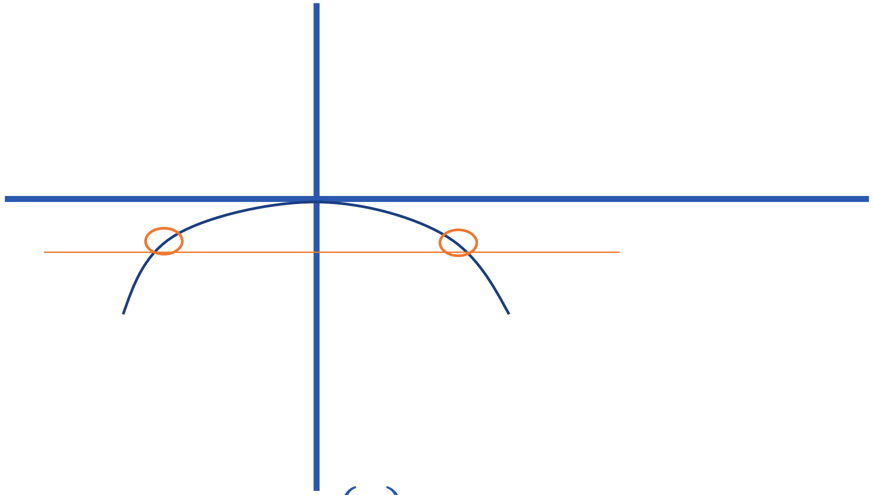
Sets, Relations and Functions

Testing of Functions

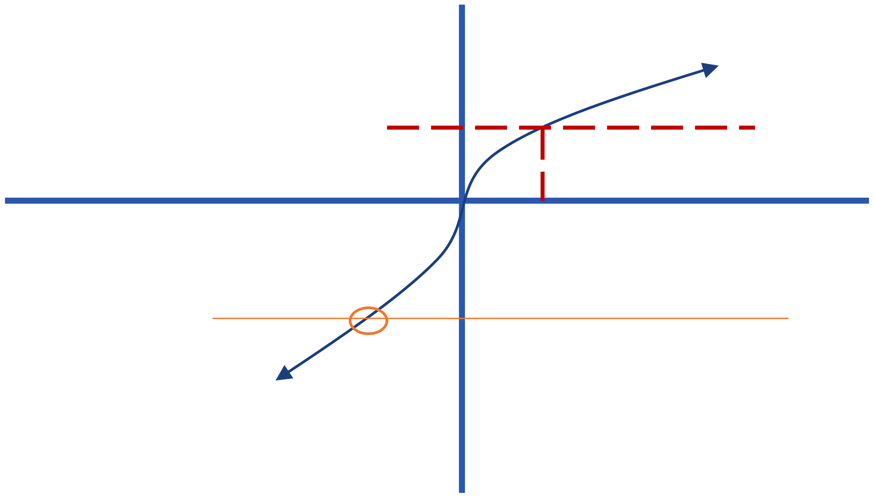
Horizontal Line Test

(a) A horizontal Line cuts the graph at two (or more) points. So, it is not a one – one function.

(b) No horizontal Line cuts the graph at more than one point. So, it is a one – one function.



(a)

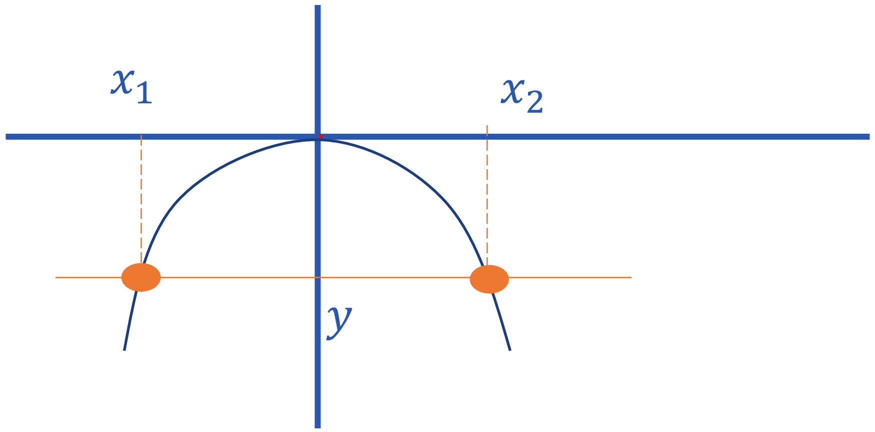


(b)

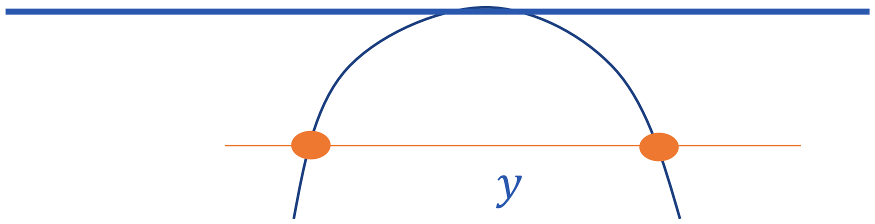
Sets, Relations and Functions

Testing of Functions

Many to one function!



Shortcut method!



If a horizontal cuts at two places, many to one function

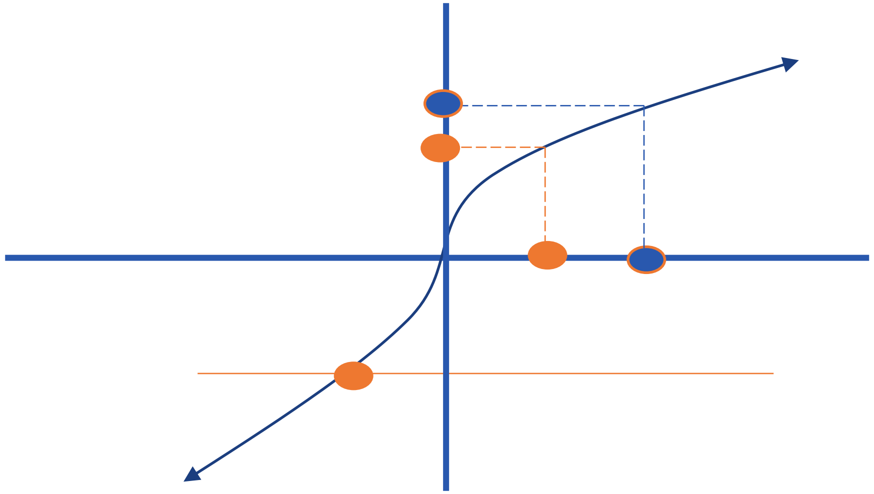
Take any y, you see both x1 and x2 likes this

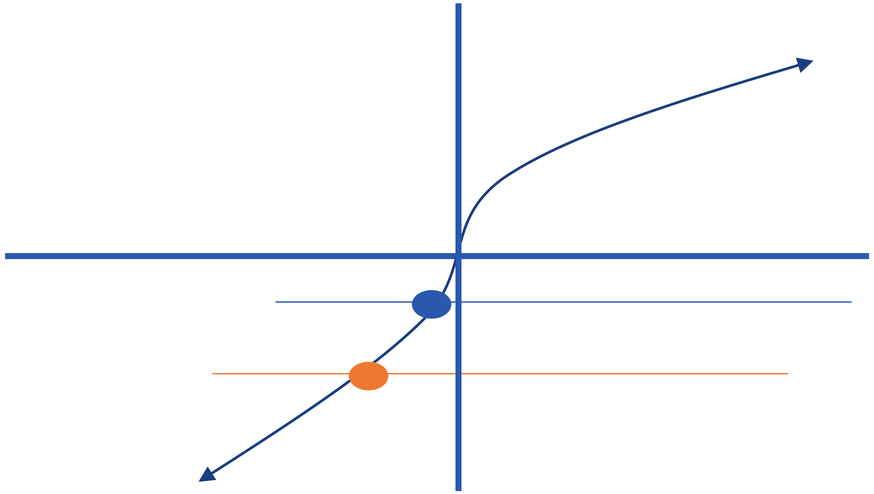
Sets, Relations and Functions

Testing of Functions

$One\ to\ one\ function!$

$Shortcut\ method!$





Take any y, you see only one $x1$ likes this

If no horizontal Line cuts the graph at more than one point. it is a one – one function.

Sets, Relations and Functions

Classification of Functions

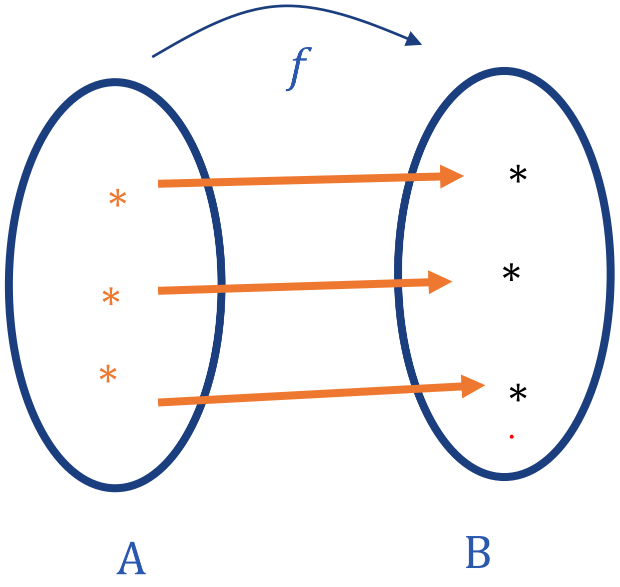
Surjectivity or onto

If range = codomain, then the function is onto (or surjective), otherwise, it is into.

from this slide for class 2

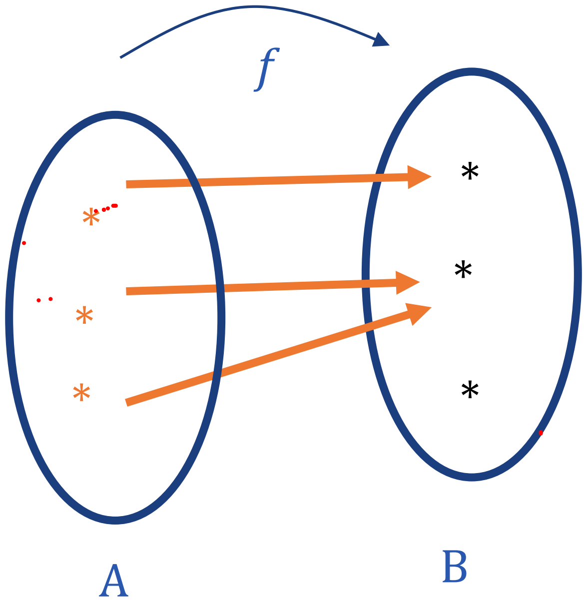
OTS

90% of students forget this after few days



Surjective (onto)

We shd learn range to understand this



into

A is domain B is codomain

$f: A \to B$

Sets, Relations and Functions

Functions

Now answer this after thinking

$\sqrt{-3}$ is defined or not? $\sqrt{-3}$ is not defined.

$\sqrt{0}\ is\ defined$ $\sqrt{0} = 0$

$\sqrt{-1}$ is defined?

Sets, Relations and Functions

Functions

$g(x) = \sqrt{x-2}$

We are given a function, but we are not given the domain

$f: A \rightarrow B$ where A is domain and B is codomain

Now tell me for what x values this is defined?

✗ Except at $x = 2$, this is defined! Yes or No $---NO.$

x belongs to all Real values except 2

Here the function $g(x)$ is defined for $x \in [2, \infty)$ $[2, \infty)$

$g(x) = \sqrt{x-2}\ :\ x \in [2, \infty)$

Sets, Relations and Functions

Domain of a Function

Domain:

Domain is primarily a collection of values of x for which the function $y = f(x)$ is defined.

We will see how to find the Domain of a given function

Sets, Relations and Functions

Domain of a Function

Consider the following

$(a)\ y = \dfrac{1}{x-4} \rightarrow x \in \mathbb{R}\backslash\{4\}$

$(b)\ y = \sqrt{x-3} \rightarrow x \in [3, \infty)$

We can find the domain from the function

$y = \sqrt{x-3}$

Sets, Relations and Functions

Rules to find Domain

(a) $\frac{1}{*}$ is defined if the denominator $* \neq 0$

(b) $\sqrt{*}$ is defined if the value of $* \geq 0$

(c) $\log_b a$ is defined if the value of a > 0 and $b > 0, b \neq 1$

(d) Domain of $f(x) + g(x)$ is the intersection of domains of $f(x)$and $g(x)$

Sets, Relations and Functions

Domain of a Function

Next Class

Sets, Relations and Functions

Domain of a Function

Find the domain of the following.

$$(a)\ y = \frac{1}{x^2 - 9}$$

$$(b)\ y = \sqrt{x^2 - 9}$$

$$(c)\ y = \frac{1}{\sqrt{x^2 - 9}}$$

$$(d)\ y = \log(x^2 - 9)$$

$$(e)\ y = \log(x - 9)$$

Sets, Relations and Functions

Domain of a Function

Find the domain of the following.

$(a)\ y = \dfrac{1}{x^2 - 9}$

y is defined if

$x^2 - 9 \neq 0$

$(x+3)(x-3) \neq 0$

$x \neq -3, x \neq 3$

$x \in R \backslash \{-3, 3\}$

Denominator shd not be zero

$x^2 - 9$ *shd not be zero*

x *shd not be* $= \pm 3$

We can also write $x \in \mathbb{R} \backslash \{-3, 3\}$

Sets, Relations and Functions

Domain of a Function

Find the domain of the following.

$(b)\ y = \sqrt{x^2 - 9}$

y is defined if

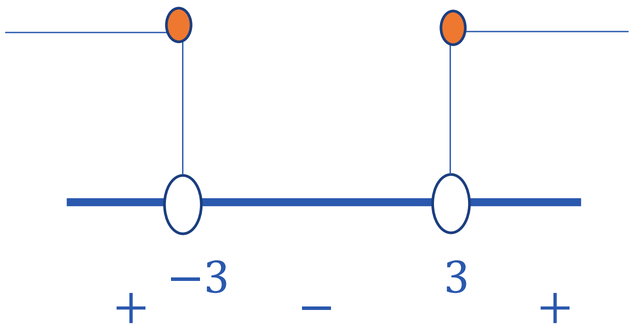
$x^2 - 9 \geq 0$

$(x+3)(x-3) \geq 0$

$x \in (-\infty, -3] \cup [3, \infty)$

$x^2 - 9$ shd be greater than or equal to zero

Hence $x^2 - 9 \geq 0$



Sets, Relations and Functions

Domain of a Function

Find the domain of the following.

$(c)\ y = \dfrac{1}{\sqrt{x^2 - 9}}$

y is defined if

$x^2 - 9 > 0$

$(x + 3)(x - 3) > 0$

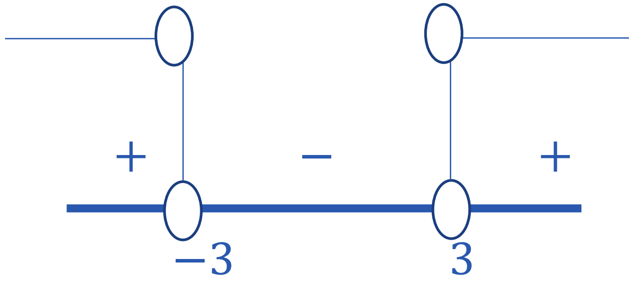
$x \in (-\infty, -3) \cup (3, \infty)$

First Root should be considered.

Root exists only if $x^2 - 9 \geq 0$

Next Denominator shd not be zero

Hence $x^2 - 9 > 0$



Sets, Relations and Functions

Domain of a Function

Find the domain of the following.

$(d)\ y = \log(x^2 - 9)$

y is defined if

$x^2 - 9 > 0$

$(x+3)(x-3) > 0$

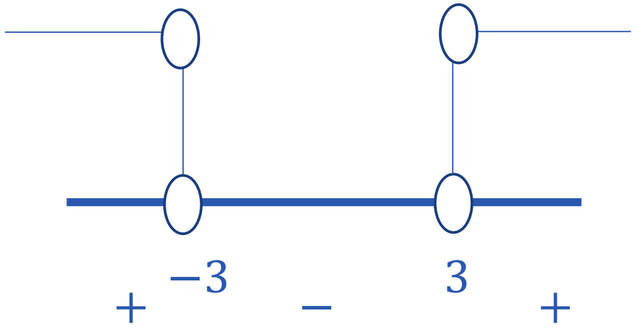
$x \in (-\infty, -3) \cup (3, \infty)$

$\log_a b$ Here $a = 10$

log will be happy when a is positive and not equal to 1

log will be happy when b is positive

Hence $x^2 - 9 > 0$



Sets, Relations and Functions

Domain of a Function

Find the domain of the following.

$(e)\ y = \log(x-9)$

y is defined if

$x - 9 > 0$

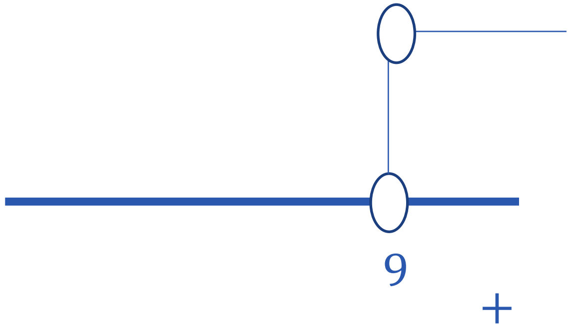
$x \in (9, \infty)$

$\log_a b$ Here $a = 0$

log will be happy when a is positive and not equal to 1

log will be happy when b is positive

Hence $x^2 - 9 > 0$



Sets, Relations and Functions

Domain of a Function

Find the domain of the following.

$(a)\ y = \log_x(4 - x^2)$

$(b)\ y = \sqrt{x^2 - 4} + \log_{10}(x - 5)$

$\log_a b$

log will be happy when a is positive and not equal to 1

log will be happy when b is positive

Sets, Relations and Functions

Domain of a Function

Find the domain of the following.

$(a)\ y = \log_x(4 - x^2)$

here base is x

$\therefore x > 0\ and\ x \neq 1$

Also, b should be positive

$\therefore 4 - x^2 > 0$

$\rightarrow x^2 - 4 < 0$

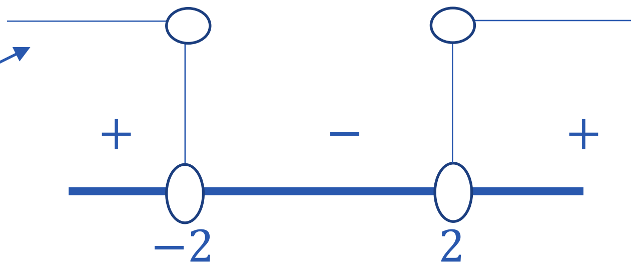
$(x - 2)(x + 2) < 0$

Combining all the three conditions

$\log_a b$

log will be happy when a is positive and not equal to 1

log will be happy when b is positive



Sets, Relations and Functions

Domain of a Function

Combining all the three conditions

$x > 0 \text{ and } x \neq 1$

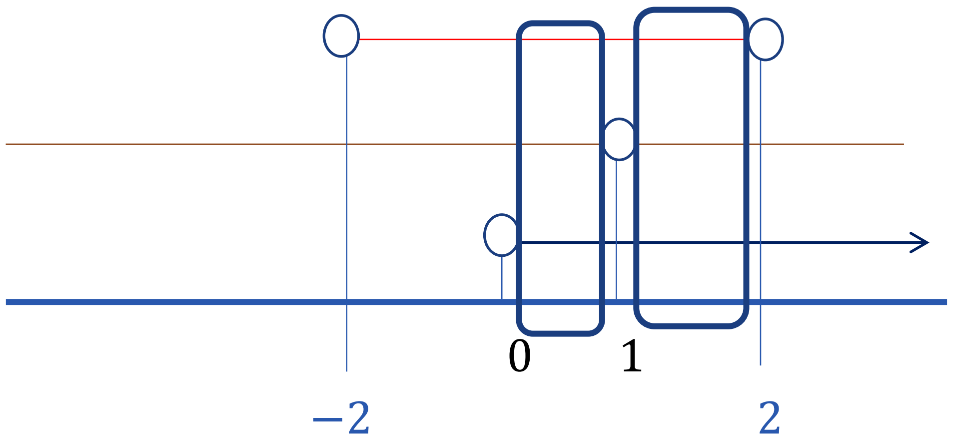
$(x-2)(x+2) < 0$

Now see the common parts in all the three

Sol: $x \in (0,2)\setminus\{1\}$

How do you intersect three things

Plot $x > 0$, plot $x \neq 1$, plot $-2 < x < 2$



Sets, Relations and Functions

Domain of a Function

Find the domain of the following.

$(b)\ y = \sqrt{x^2 - 4} + \log_{10}(x - 5)$

When more than one conditions come,

we shd consider all the conditions

$\sqrt{x^2 - 4}\ \ is\ defined\ if\ x^2 - 4 \geq 0$

$(x - 2)(x + 2) \geq 0$

$-2 \leq x \leq 2$

$\log_{10}(x - 5)\ is\ defined\ if\ (x - 5) > 0$

$x > 5$

$f(3) = \sqrt{9 - 4} + \log_{10}(3 - 5)$

$If\ I\ take\ x = 3, then \sqrt{x^2 - 4}\ \ will\ be\ defined.$

$Yes\ or\ No?$

$but, \log_{10} x - 5\ \ will\ be\ undefined. Yes\ or\ No?$

Sets, Relations and Functions

Module Name

$x^2 - 4 \geq 0 \; and \; x - 5 > 0$ and means simultaneous

$x^2 - 4 \geq 0 \cap x - 5 > 0$

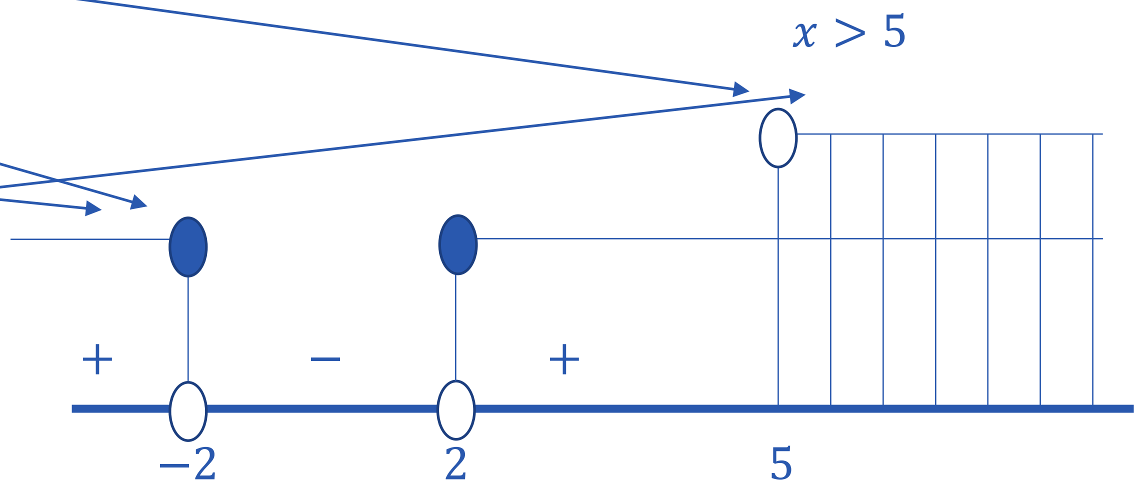
$(x-2)(x+2) \geq 0$

$-2 \leq x \leq 2$

$\cap$

$x - 5 > 0$

$x \in (5, \infty)$



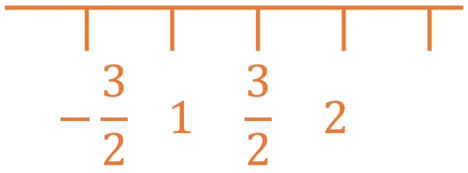
Sets, Relations and Functions

Domain of a Function

Find the domain of the following.

$$y = \sqrt{\frac{(x-1)(x-2)}{(9-4x^2)}}$$

More and more functions you learn, more and more we need to learn about domain



When i do this, tell me you will feel that that is obvious.

y is defined if

$$\geq 0, x \neq \frac{3}{2}$$

$$\frac{(x-1)(x-2)}{(9-4x^2)} \geq 0$$

Only one condition here.

Dont make separate conditions for each term

$$\frac{(x-1)(x-2)}{(2x-3)(2x+3)} \leq 0$$

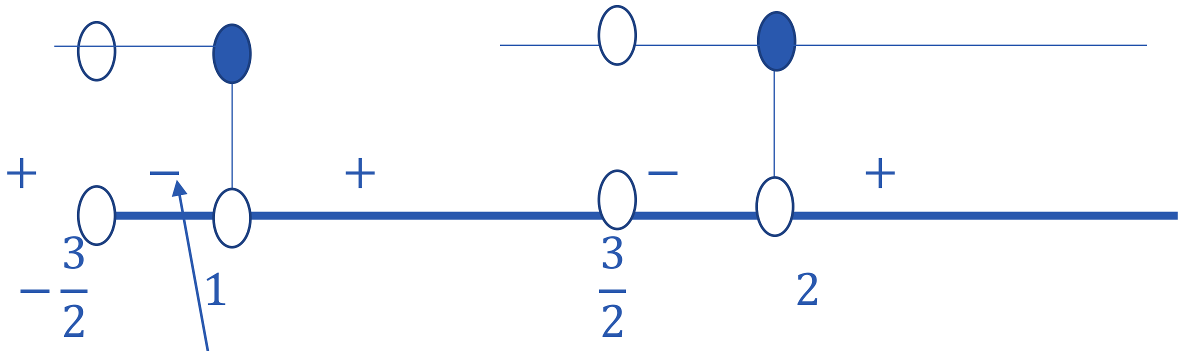
Now everything is considered

Sets, Relations and Functions

Domain of a Function

$$\frac{(x-1)(x-2)}{(2x-3)(2x+3)} \leq 0$$

$$x \in \left(-\frac{3}{2}, 1\right] U \left(\frac{3}{2}, 2\right]$$



At $x = 0, f(x)$ is negative

Sets, Relations and Functions

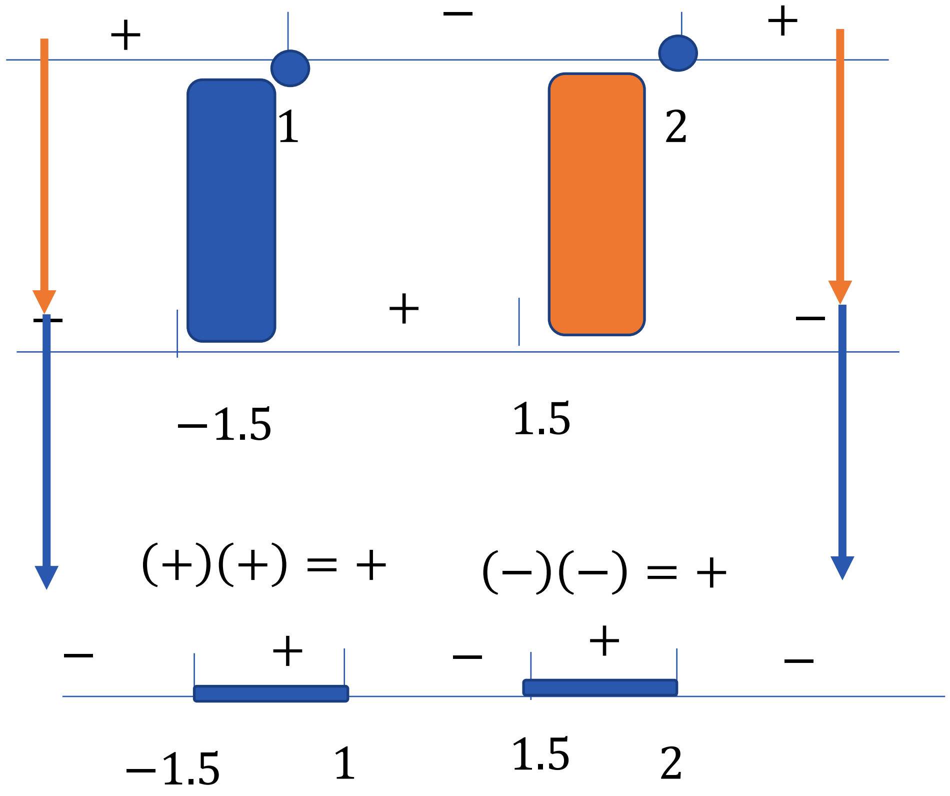
Domain of a Function

$$(x-1)(x-2) \geq 0$$

$$(3-2x)(3+2x) > 0$$

$$\frac{(x-1)(x-2)}{(3-2x)(3+2x)} > 0$$

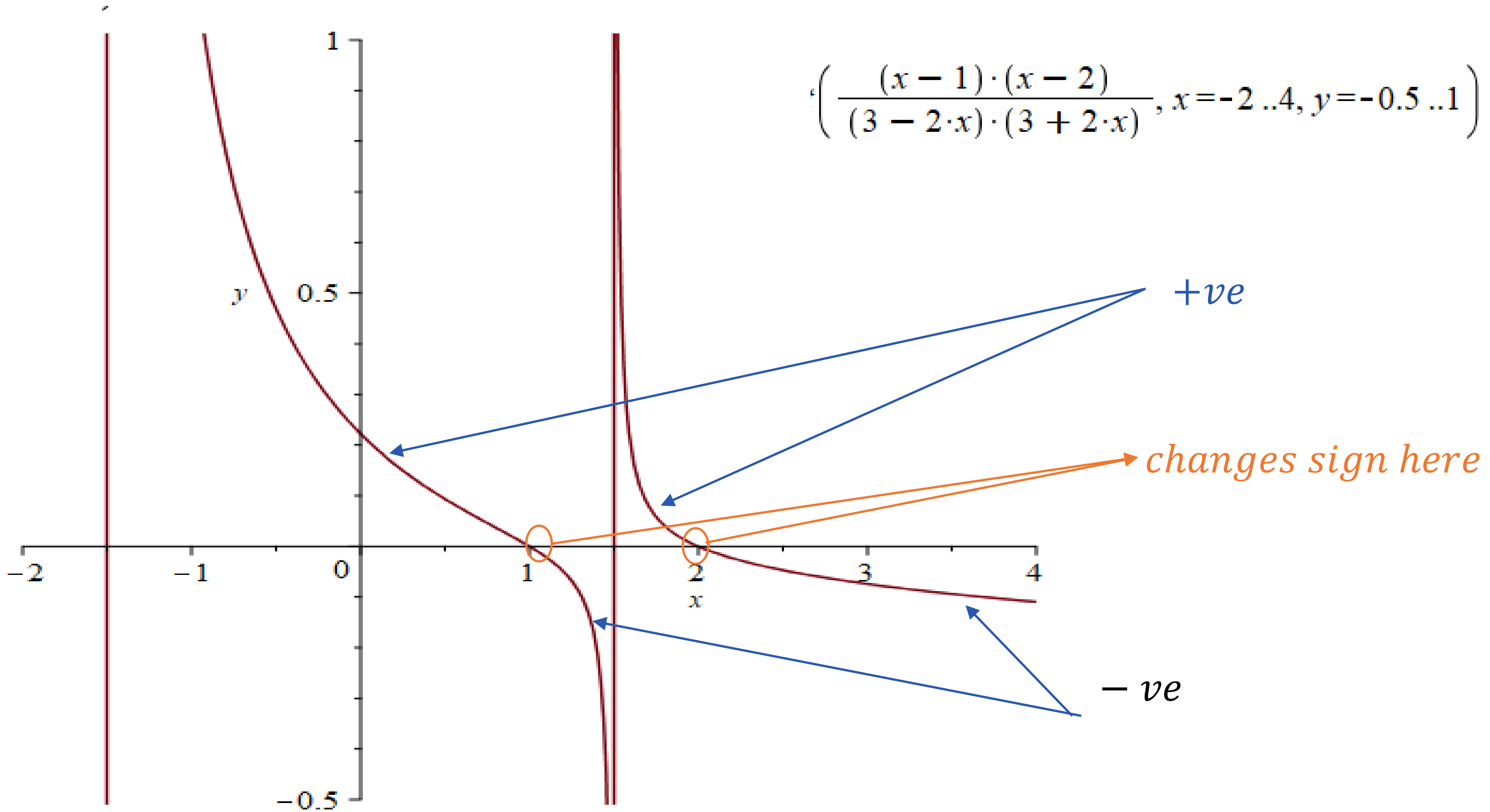
$$x \in \left(-\frac{3}{2}, 1\right] U \left(\frac{3}{2}, 2\right]$$



Sets, Relations and Functions

Domain of a Function

$$x \in \left(-\frac{3}{2}, 1\right] U \left(\frac{3}{2}, 2\right]$$



Sets, Relations and Functions

Domain of a Function

Find the domain of the following.

$$y = \sqrt{1 - \sqrt{1 - x^2}}$$

First let us make the inner square root happy!

y is defined if $1 - x^2 \geq 0$

Next let us make the outer square root happy!

$$1 - \sqrt{1 - x^2} \geq 0$$

Both shd be considered

Order is not important because simultaneously they shd be satisfied

$$1 - x^2 \geq 0 \text{ and } 1 - \sqrt{1 - x^2} \geq 0$$

Sets, Relations and Functions

Domain of a Function

$1 - x^2 \geq 0$ and $1 - \sqrt{1 - x^2} \geq 0$

$1 - x^2 \geq 0$

$x^2 - 1 \leq 0$

$(x + 1)(x - 1) \leq 0$

$-1 \leq x \leq 1$

Now the second one needs a little more concentration from your end.

$1 - \sqrt{1 - x^2} \geq 0$

$1 \geq \sqrt{1 - x^2}$

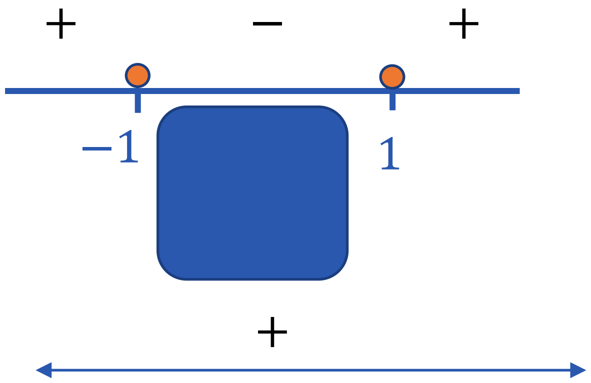
Squaring on both sides

$1 \geq 1 - x^2$

$x^2 \geq 0 \rightarrow x \in R$

$-1 \leq x \leq 1 \ \cap x^2 \geq 0 \rightarrow x \in R$

Sol: $[-1, 1]$



Sets, Relations and Functions

Domain of a Function

$\sqrt{1-x^2} \leq 1$ is always less than or equal to one!

Is $a \geq b \Rightarrow a^2 \geq b^2$ always ?

No, it is not always

$a \geq b \Rightarrow a^2 \geq b^2$ *only if a and b* ≥ 0

$5 \succ 10$

$5^2 < (-10)^2$

$-10 \succ -20$

$100 < 400$

$a \geq b \Rightarrow a^2 \geq b^2$ *if a and b* ≥ 0

Sets, Relations and Functions

Domain of a Function

$a \geq b$

$a^2 \geq b^2$

if a and $b \geq 0$

$25 > 16$

$\therefore 5 > 4$

$a^2 \leq b^2$

if a and $b < 0$

$16 < 25$

$\therefore -4 > -5$

if one is + ve and one is − ve

Here anyting can happen

$if\ a > 0, b < 0$

$5 > -2$ $\quad$ $5 > (-10)$

$25 > 4$ $\quad$ $25 < 100$

Modulus is therefore important

$\sqrt{a\ number}$ is always positive

$\sqrt{25} = +5$ only; *not* ± 5

Sets, Relations and Functions

Domain of a Function

Solve $\sqrt{x^2 - 4} < 3$ for x

Very important . We learn a new thing here.

<u>Squaring both sides</u>

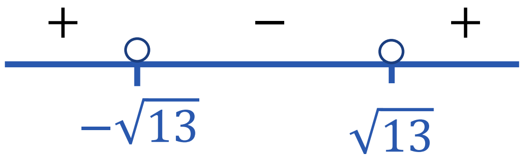
$x^2 - 4 < 9$

$x^2 - 13 < 0$

$(x - \sqrt{13})(x + \sqrt{13}) < 0$

$-\sqrt{13} < x < \sqrt{13}$

Wrong! Why?



At $x = 1 \rightarrow \sqrt{x^2 - 4} = \sqrt{1 - 4} \nless 3$

$\sqrt{-3}$ is undefined

Sets, Relations and Functions

Domain of a Function

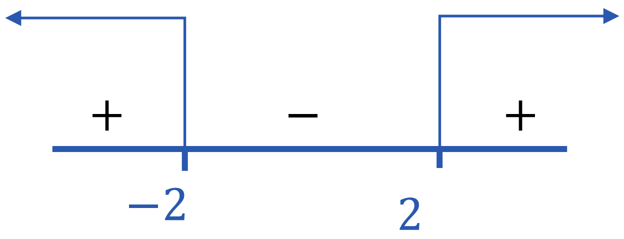
<u>Here x does not know that there is a root here. We have removed the root in the first step!</u>

If there is a root in the problem and you square it to solve, then such problems will arise.

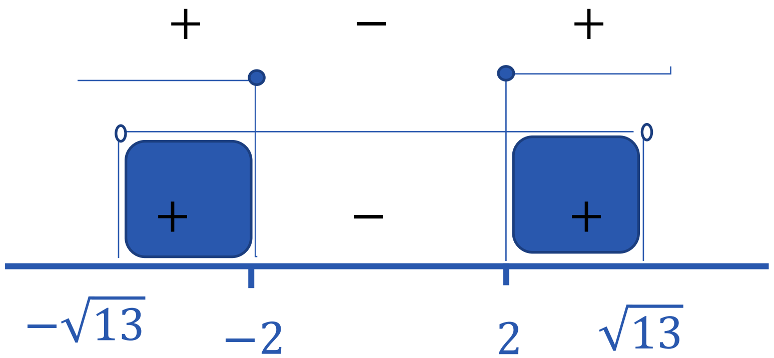
You must take care of this differently

Also, $x^2 - 4 \geq 0$

$(x+2)(x-2) \geq 0$



Taking the common (intersection)



Sol: $\left(-\sqrt{13}, -2\right] U \left[2, \sqrt{13}\right)$

$for \sqrt{x^2 - 4} < 3$

Sets, Relations and Functions

Domain of a Function

Solve $\sqrt{x^2 - 4} < 2$ for x

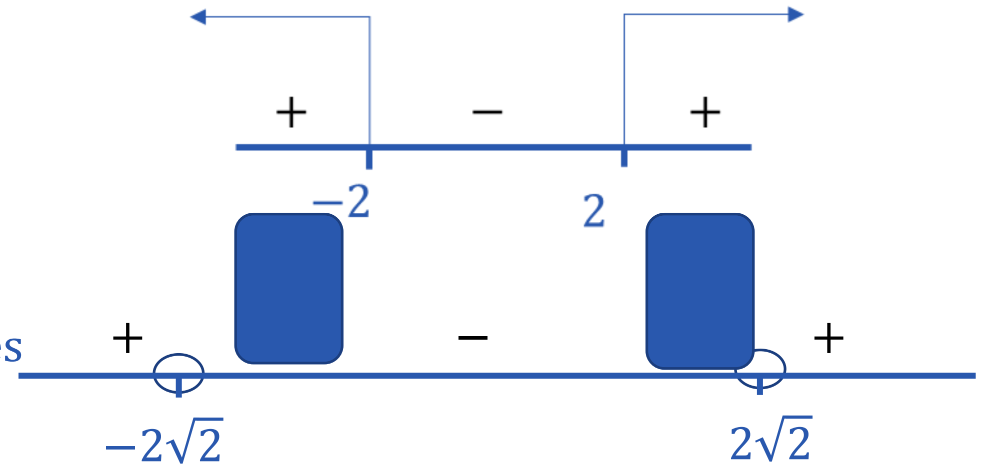
Square root inside shd be positive ,

and outside also shd be positive

$x^2 - 4 \geq 0$

$(x+2)(x-2) \geq 0$

$\therefore x \in (-\infty, -2] \cup [2, \infty)$



$\sqrt{x^2 - 4} < 2$ Squaring both sides

$x^2 - 4 < 4$

$\sqrt{8} = 2\sqrt{2}$

$x^2 < 8$ $\therefore -\sqrt{8} < x < \sqrt{8}$

$\therefore -2\sqrt{2} < x < 2\sqrt{2}$

Taking the intersection (common)

$x \in \left(-2\sqrt{2}, -2\right] \cup \left[2, 2\sqrt{2}\right)$

Sets, Relations and Functions

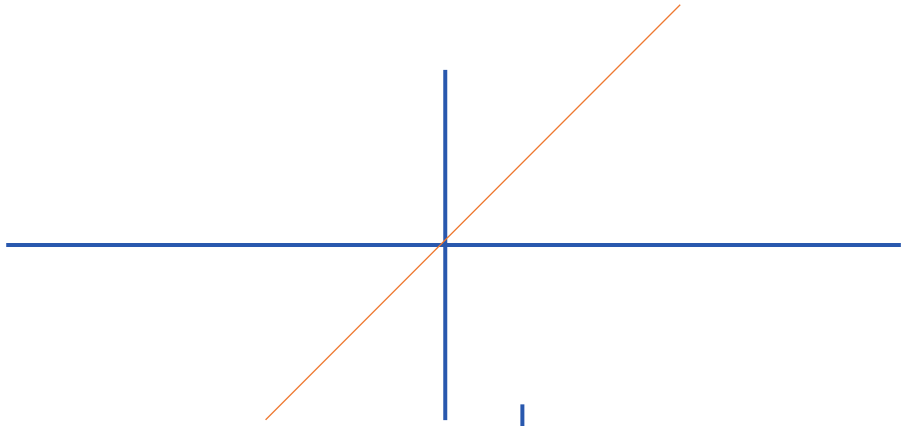
Domain of a Function

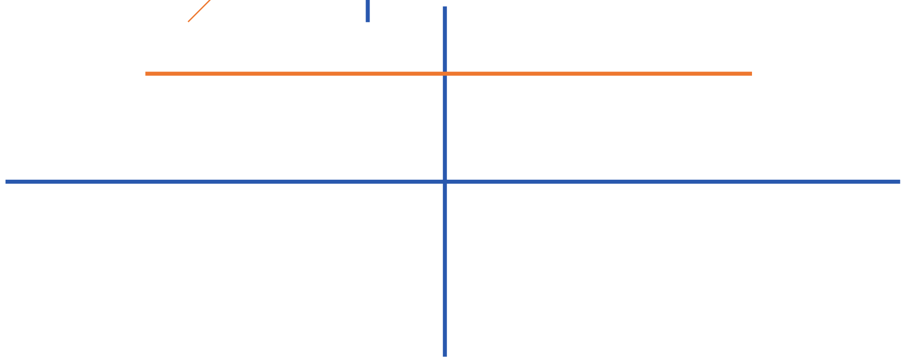
Identity function

Constant function

Polynomial function

Rational function

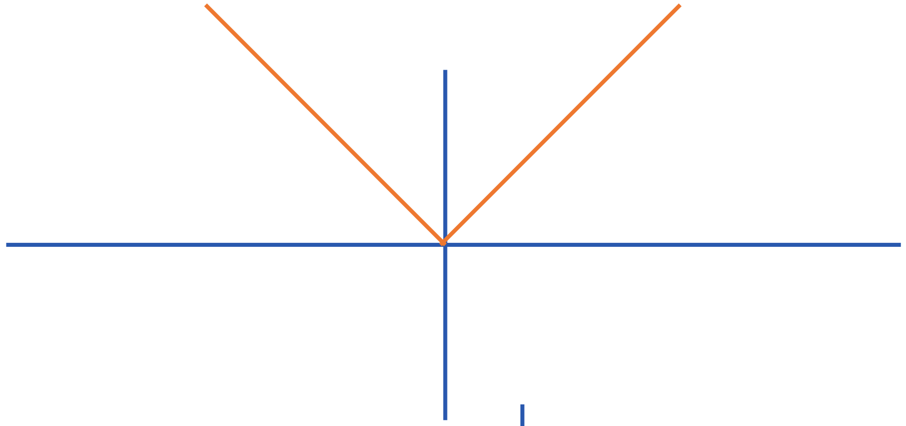




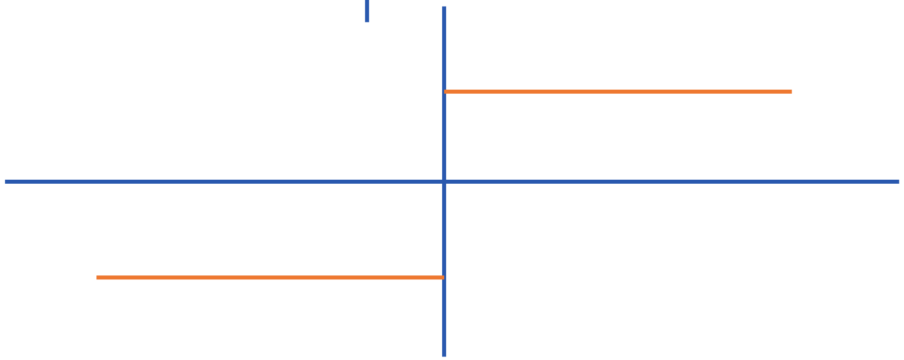
Sets, Relations and Functions

Some Functions and their Graphs

Modulus Function



Signum Function



Greatest Integer Function

Sets, Relations and Functions

Some Functions and their Graphs

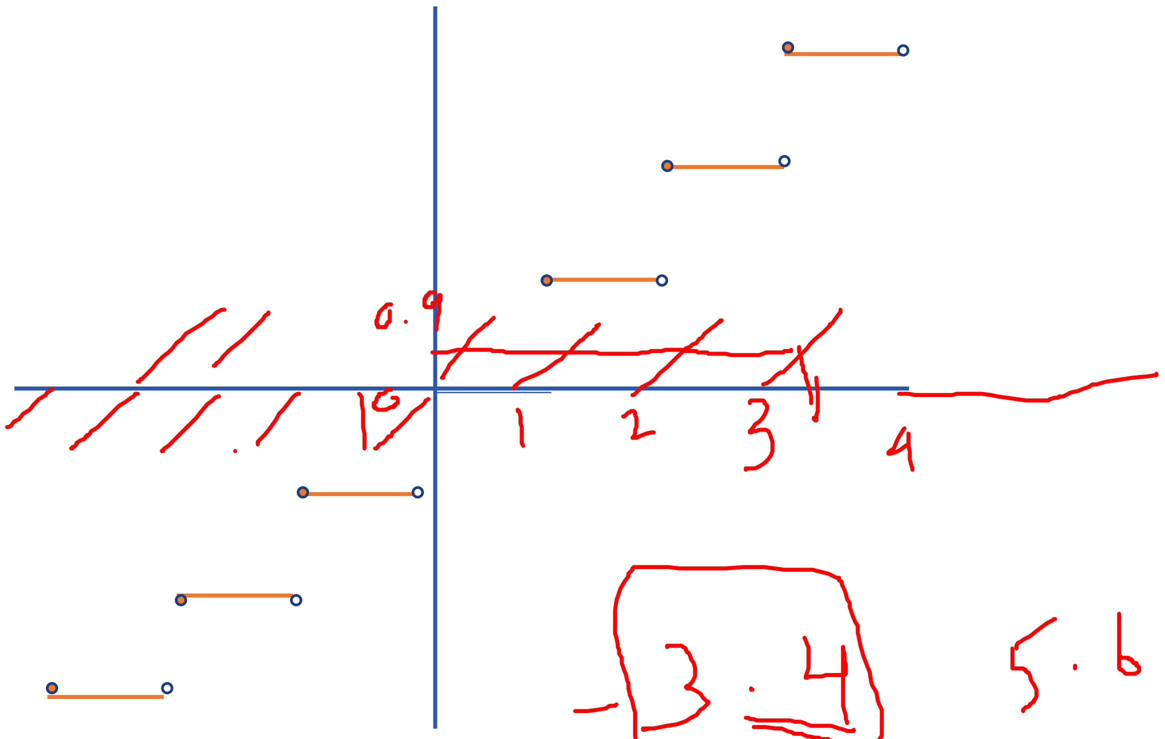
Greatest Integer Function

$-x = [x] + \{0.4\}$

3.4 =

10.8

0.1 −1 −1.2



−3.4

5.6

1 2 3 4

3.9

Sets, Relations and Functions

Range of a Function

Example

Find the range of the function $f(x) = x^2 - x - 1$ $\qquad y \in \left[-\frac{5}{4}, \infty\right)$

Find the range of the function $f(x) = x^2 + x + 1$ $\qquad y \in \left[\frac{3}{4}, \infty\right)$

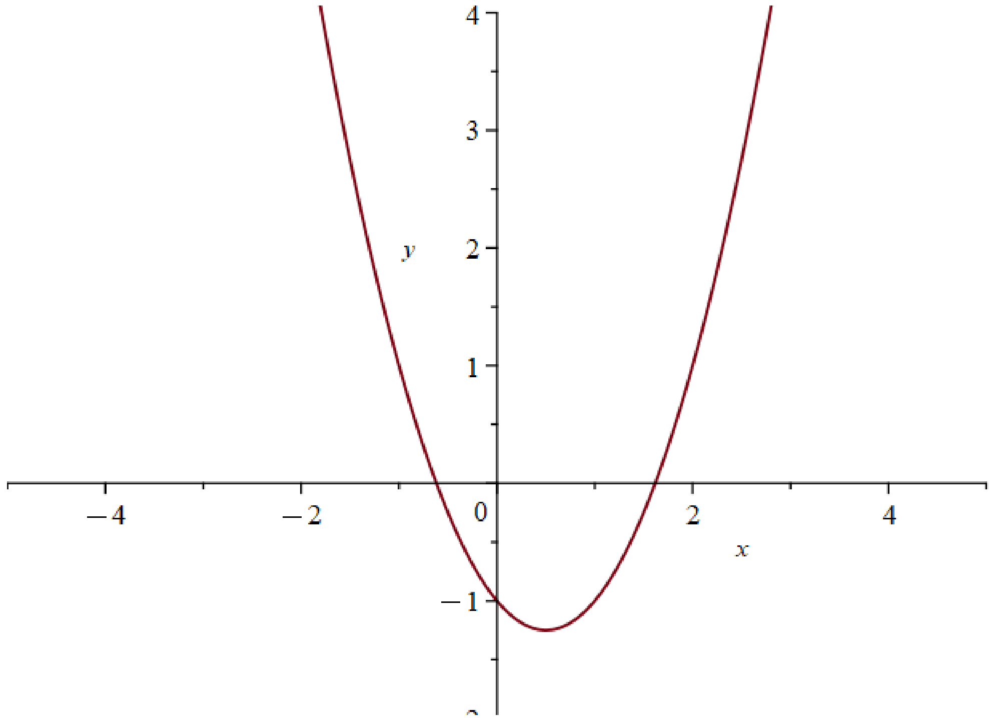
Find the range of the function $f(x) = \dfrac{x^2 + x + 2}{x^2 + x + 1}$ $\qquad y \in \left(1, \frac{7}{3}\right]$

Sets, Relations and Functions

Range of a Function

Example

Find the range of the function $f(x) = x^2 - x - 1$



d = 0

d^2 +ve

$y \in \left[-\frac{5}{4}, \infty\right)$

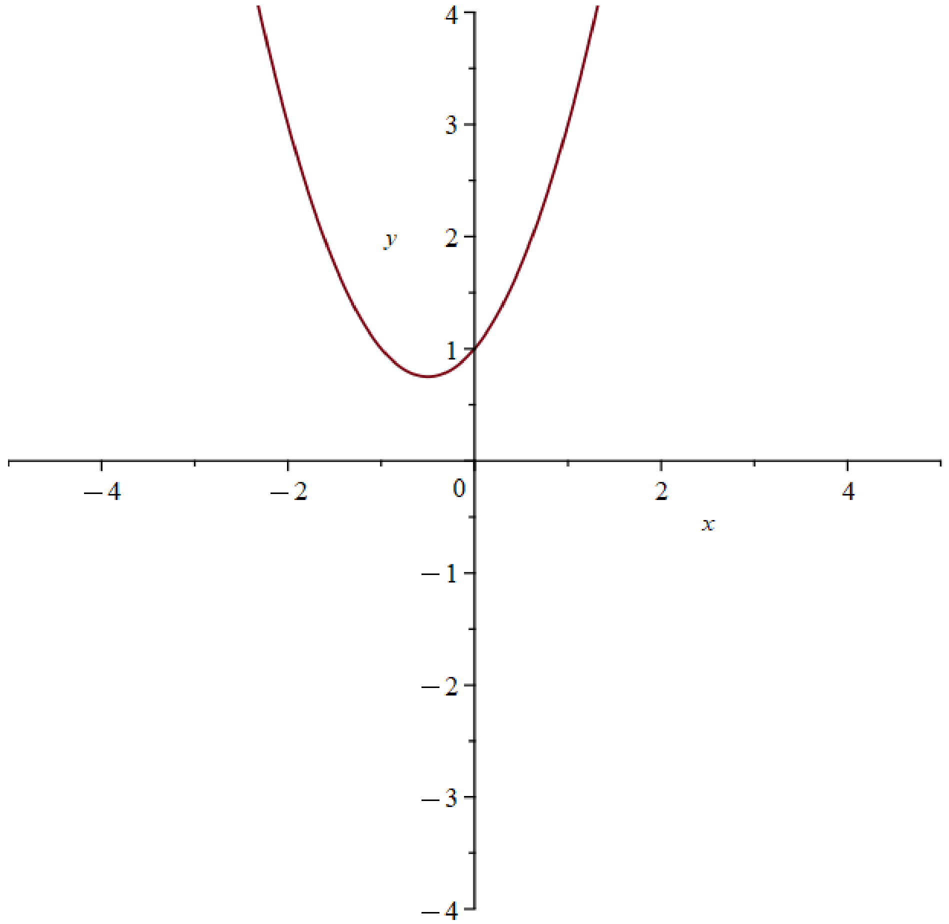
Sets, Relations and Functions

Range of a Function

Example

Find the range of the function $f(x) = x^2 + x + 1$

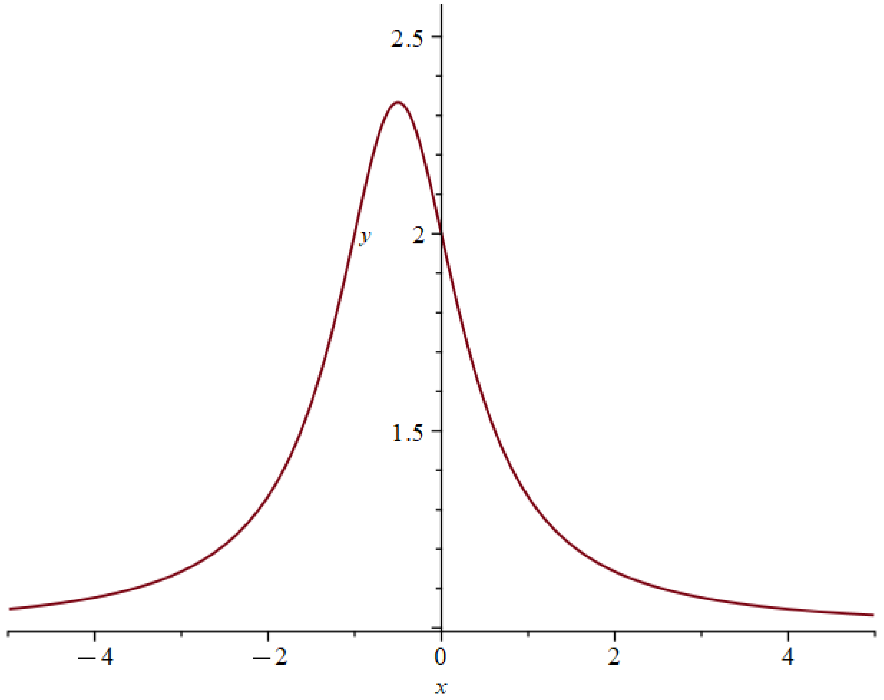
$$y \in \left[\frac{3}{4}, \infty\right)$$



Range of a Function

Example

Find the range of the function $f(x) = \dfrac{x^2+x+2}{x^2+x+1}$



$$y \in \left(1, \frac{7}{3}\right]$$

Sets, Relations and Functions

Range of a Function

Example

$Find\ the\ range\ of\ the\ function\ f(x) = x^2 - x - 1$

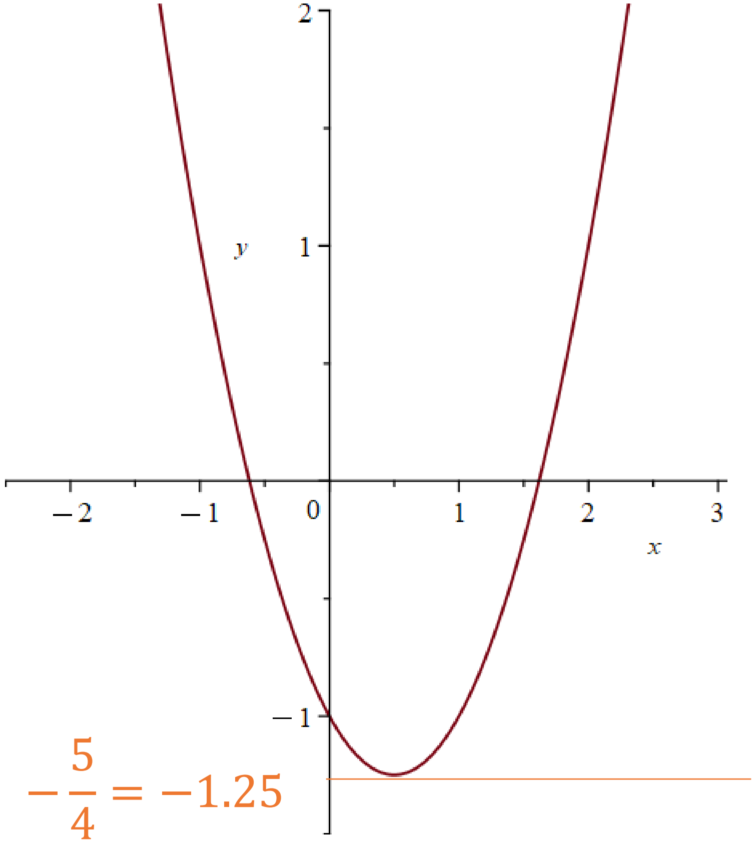
$$y = x^2 - x - 1$$

$$\frac{dy}{dx} = 2x - 1 = 0 \rightarrow x = \frac{1}{2}$$

$$\frac{d^2y}{dx^2} = 2, Positive\ constant\ \therefore y\ is\ minimum\ at\ x = \frac{1}{2}$$

$$y\left(\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{2} - 1 = -\frac{5}{4}$$

$$y \in \left[-\frac{5}{4}, \infty\right)$$



$$-\frac{5}{4} = -1.25$$

Sets, Relations and Functions

Range of a Function

Example

$Find\ the\ range\ of\ the\ function\ f(x) = x^2 + x + 1$

$y = x^2 + x + 1$ $\quad$ $Domain\ is\ x \in R$

$$\frac{dy}{dx} = 2x + 1 = 0 \rightarrow x = -\frac{1}{2}$$

$$\frac{d^2y}{dx^2} = 2, Positive\ constant\ \therefore y\ is\ minimum\ at\ x = -\frac{1}{2}$$

$$y\left(-\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}$$

$$y \in \left[\frac{3}{4}, \infty\right)$$

Aliter:

$$y = \left(x + \frac{1}{2}\right)^2 + 1 - \frac{1}{4}$$

0

$$y_{max} = \left(\infty + \frac{1}{2}\right)^2 + 1 - \frac{1}{4} = \infty$$

$$y_{min} = 0 + 1 - \frac{1}{4} = \frac{3}{4}$$

$$\therefore y \in \left[\frac{3}{4}, \infty\right)$$

Sets, Relations and Functions

Range of a Function

Example

$$Find\ the\ range\ of\ the\ function\ f(x) = \frac{x^2+x+2}{x^2+x+1}$$

$$y = \left(1, \frac{7}{3}\right]$$

$$y = \frac{x^2+x+2}{x^2+x+1}$$

$$y(x^2+x+1) = x^2+x+2$$

$$x^2(y-1) + x(y-1) + y - 2 = 0$$

We know Domain of x is R.

$$Discriminant, D \geq 0$$

$$(y-1)^2 - 4(y-1)(y-2) \geq 0$$

$$-3y^2 - 10y + 7 \geq 0$$

$$(3y-7)(y-1) \geq 0$$

Now verify the end points for $y = 1$

$$x^2+x+2 \neq x^2+x+1$$

So 1 is excluded

$$So, y \in \left(1, \frac{7}{3}\right]$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{(b^2-4ac)}}{2a}$$

Sets, Relations and Functions

Range of a Function

Example

Find the range of the function $f(x) = \dfrac{x^2 + x + 2}{x^2 + x + 1}$

$$y = \left(1, \frac{7}{3}\right]$$

$$y = \frac{x^2 + x + 2}{x^2 + x + 1} = \frac{(x^2 + x + 1) + 1}{(x^2 + x + 1)}$$

$$y = 1 + \frac{1}{x^2 + x + 1}$$

$$y = 1 + \frac{1}{\left(x + \frac{1}{2}\right)^2 + 1 - \frac{1}{4}}$$

$$y = 1 + \frac{1}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$y_{max} = +1 - \frac{1}{0 + \frac{3}{4}} = \frac{7}{3}$$

$$y_{min} = +1 + \frac{1}{\infty} = 1$$

$$So, y \in \left(1, \frac{7}{3}\right]$$

Sets, Relations and Functions

Subsets

$If\ A\ is\ a\ set\ with\ n\ elements, total\ number\ of\ subsets\ possible = 2^n$

$Number\ of\ elements\ in\ power\ set of\ a\ set\ A = 2^{n(A)}$

$A = \{1,2,3\}$

$Number\ of\ subsets = 2^3 = 8$

$Subsets = \{.\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

$Powerset, P(A) = \{\{.\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Sets, Relations and Functions

Relations

$Equivalence\ relations \qquad R \quad A \to A$

$A\ relation\ is\ equivalence\ relation\ only\ if\ it\ is\ reflexive, symmetric\ and\ transitive$

$A\ relation\ is\ reflexive$

$if\ \forall a \in A, (a, a) \in R$

$(a, a) \in R\ and\ aRa\ both\ are\ same$

$A\ relation\ is\ symmetric\ if$

$for\ any\ a, b \in A,$

$(a, b) \in R \to (b, a) \in R$

$A\ relation\ is\ transitive\ if$

$for\ any\ a, b, c \in A,$

$(a, b) \in R\ , (b, c) \in R \to (a, c) \in R$

$A = \{1, 2, 3\}$

$(1,1) \in R$

$(2,2) \in R$

$(3,3) \in R$

$Note: \forall$

Sets, Relations and Functions

Relations

$A\ relation\ is\ reflexive$

$if\ \forall a \in A, (a, a) \in R$

$(a, a) \in R\ and\ aRa\ both\ are\ same$

$A = \{1, 2, 3\}$ $Note{:}\ \forall$

$(1,1) \in R$ $(2,2) \in R$ $(3,3) \in R$

$A\ relation\ is\ symmetric\ if$

$for\ any\ a, b \in A,$

$(a, b) \in R \rightarrow (b, a) \in R$

$A\ relation\ is\ transitive\ if$

$for\ any\ a, b, c \in A,$

$(a, b) \in R\ , (b, c) \in R \rightarrow (a, c) \in R$

$A\ relation\ is\ NOT\ reflexive$

$\exists\ a \in A, (a, a) \notin R$

$\exists$ *is read as " there exists"*

$A\ relation\ is\ NOT\ symmetric\ if$

$\exists\ a, b \in A,$

$(a, b) \in R, but\ (b, a) \notin R$

$A\ relation\ is\ NOT\ transitive\ if$

$\exists\ a, b, c \in A,$

$(a, b), (b, c) \in R, but\ (a, c) \notin R$

Sets, Relations and Functions

Relations

Example

Check for reflexivity, symmetry and trastivity

$aRb\ iff\ b\ is\ divisible\ by\ a, a, b \in N$ (OR) $R = \{(a, b): b\ is\ divisible\ by\ a; a, b \in N\}$

A relation is NOT reflexive

$\exists\ a \in A, (a, a) \notin R$

$\exists$ *is read as " there exists"*

$\exists\ 1, 2 \in NA,$

$(1,1) \in R, (2,2) \in R$

A relation is NOT symmetric if

$\exists\ a, b \in A,$

$(a, b) \in R, but\ (b, a) \notin R$

$\exists\ 1,2 \in N, (1,2) \in R$

$but (2,1) \notin R$

1 is not divisible by 2

A relation is NOT transitive if

$\exists\ a, b, c \in A,$

$(a, b), (b, c) \in R, but\ (a, c) \notin R$

$\exists\ a, b, c \in N,$

$(a, b) \in R \rightarrow b = ma$

$(b, c) \in R \rightarrow c = nb = nm\ a$

$\therefore (a, c) \in R$

$\therefore$ *R is reflexive and transtive but not symmetric.*

$\therefore$ *R is not equivalence relation..*

Q 1.

JEE Mathematics – Functions

Find the domain and range of the function $f(x) = |x-1|$

Option 1: $R, R-(0,1)$

Option 2: $R, R-\{3\}$

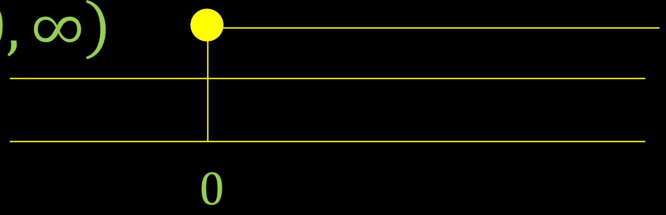
Option 3: $R, [0,\infty)$

Option 4: *none of these*

$$f(x) = |x-1| = \begin{cases} x-1 & if\ x \geq 1 \\ 1-x & if\ x < 1 \end{cases} \quad where\ x \in R$$

$$|x-1| \geq 0$$

$$\therefore Range\ is = R^+, \{0\} = [0,\infty)$$



Option 3: $R, [0,\infty)$

If $f: A \to B$ *be a function, the set A is domain of the function.*
The set B is codomain of the function. Range is subset of the codomain.

Q 2.

JEE Mathematics – Functions

Find the domain and range of the function $f(x) = |x+1| - 2$

Option 1: $R, R - (0,1)$

Option 2: $R, R - \{3\}$

Option 3: $R, [-2, \infty)$

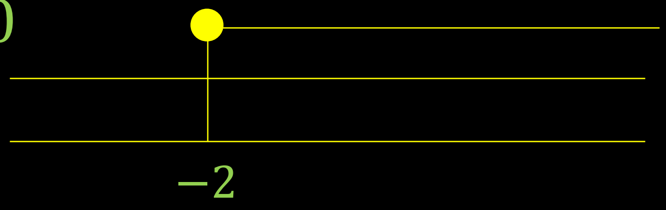
Option 4: *none of these*

Given $f(x) = |x+1| - 2$ *where* $x \in R$

$|x+1| \geq 0, \forall x \in R$

$|x+1| - 2 \geq 0 - 2 \geq 0$

Option 3: $[-2, \infty)$



If $f: A \to B$ *be a function, the set A is domain of the function.*
The set B is codomain of the function. Range is subset of the codomain.

Q 3.

JEE Mathematics – Functions

Find the domain and range of the function $f(x) = |x-1| + |x-2|, -1 \leq x \leq 3.$

Option 1: $R, R-(0,1)$

Option 2: $R, R-\{3\}$

Option 3: $[-1,3], [1,5]$

Option 4: *none of these*

$$f(x) = |x-1| + |x-2|$$

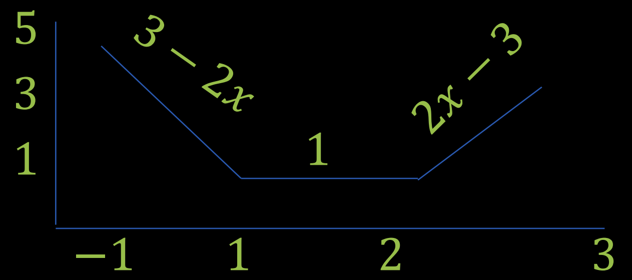
$-1 \leq x < 1, f(x) = -(x-1) - (x-2) = 3 - 2x$

$1 \leq x < 2, f(x) = (x-1) - (x-2) = 1$

$2 \leq x \leq 3, f(x) = (x-1) + (x-2) = 2x - 3$

$\therefore$ *Domain and Range are*

Option 3: $[-1,3], [1,5]$



If $f: A \rightarrow B$ *be a function, the set A is domain of the function.*
The set B is codomain of the function. Range is subset of the codomain.

Q 4.

JEE Mathematics – Functions

Find the domain and range of the function $f(x) = \frac{x^2}{1+x^2}$

Option 1: R^+ and R

Option 2: R and $[0,1)$

Option 3: $[2,5]$ and $[0,1)$

Option 4: $(2,5)$ and $(3,5)$

$$f(x) = y = \frac{x^2}{1+x^2} \rightarrow \text{domain is } R$$

$$y(1+x^2) = x^2 \rightarrow x^2(y-1) + y = 0$$

$$\rightarrow x^2 = \frac{y}{1-y} \rightarrow x = \pm\sqrt{\frac{y}{1-y}}$$

Option 2: R and $[0,1)$

Home Work

If $f: A \rightarrow B$ be a function, the set A is domain of the function.
The set B is codomain of the function. Range is subset of the function.

Q 5.

JEE Mathematics – Functions

Let $A = \{1,2,3,4\}$. *The number of functions* $f: A \to A$ *satisfying* $f(f(i)) = 1$ *for all* $1 \leq i \leq 4$ *is*

Option 1: 1

Option 2: 6

Option 3: 9

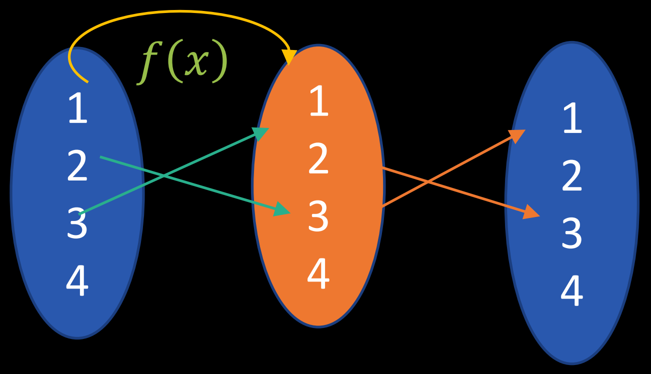
Option 4: 10

$f(2) \neq 1,2 \to f(f(2)) = f(1) \neq 1$

$f(2) \neq 1,2 \to f(f(2)) = f(2) \neq 1$

$f(3) \neq 3,1 \to f(f(3)) = f(1) = \neq 1$

$f(4) \neq 4,1 \to f(f(4)) = f(1) = \neq 1$



$f(2) = 1$ *then* $f(1) \neq 1$ $\because$ *many to one*

$f(1) = 1,1 \to f(f(1)) = 1$

$f(2) = 3$ *and* $f(3) = 1 \to f(f(2)) = f(3) = 1$

$f(2) = 4$ *and* $f(4) = 1 \to f(f(2)) = f(4) = 1$

$f(3) = 2$ *and* $f(2) = 1 \to f(f(3)) = f(2) = 1$

$f(3) = 4$ *and* $f(4) = 1 \to f(f(3)) = f(4) = 1$

$f(4) = 2$ *and* $f(2) = 1 \to f(f(4)) = f(2) = 1$

$f(4) = 3$ *and* $f(3) = 1 \to f(f(4)) = f(3) = 1$

$f(1) = 2$ *and* $f(2) = 1 \to f(f(1)) = f(2) = 1$

$f(1) = 3$ *and* $f(3) = 1 \to f(f(1)) = f(3) = 1$

$f(1) = 4$ *and* $f(4) = 1 \to f(f(1)) = f(4) = 1$

Q 6.

JEE Mathematics – Functions

Let $A = \{1,2,3,4\}$. The number of functions $f: A \rightarrow A$ satisfying $f(f(i)) = i$ for all $1 \leq i \leq 4$ is

Option 1: 1

Option 2: 6

Option 3: 9

Option 4: 10

$f(1) = 1, 1 \rightarrow f(f(1)) = 1$

$f(1) = 2 \text{ and } f(2) = 1 \rightarrow f(f(1)) = f(2) = 1$

$f(1) = 3 \text{ and } f(3) = 1 \rightarrow f(f(1)) = f(3) = 1$

$f(1) = 4 \text{ and } f(4) = 1 \rightarrow f(f(1)) = f(4) = 1$

$f(2) = 2 \rightarrow f(f(2)) = f(2) = 2$

$f(2) = 3 \text{ and } f(3) = 2 \rightarrow f(f(2)) = f(3) = 2$

$f(2) = 4 \text{ and } f(4) = 2 \rightarrow f(f(2)) = f(4) = 2$

$f(3) = 3 \rightarrow f(f(3)) = f(3) = 3$

$f(3) = 4 \text{ and } f(4) = 3 \rightarrow f(f(3)) = f(4) = 3$

$f(4) = 4 \rightarrow f(f(4)) = f(4) = 4$

Q 7.

JEE Mathematics – Functions

Let $f:\{1,2,3\} \to \{1,2,3\}$ *be a function. If the number of functions* $g:\{1,2,3\} \to \{1,2,3\}$ *such that* $f(x) = g(x)$ *for atleast one* $x \in \{1,2,3\}$ *is* k, *then* $(k-10)$ *is equal to*

Option 1: 19

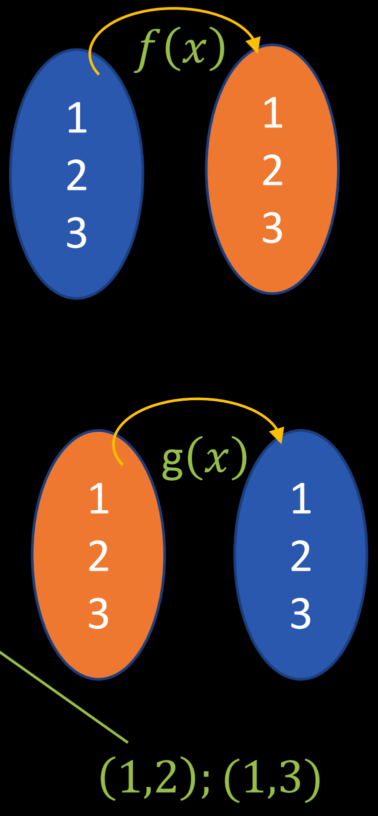
Option 2: 9

Option 3: 8

Option 4: 17

$$n\,(f: A \to B) = 3^3 = 27$$

$$n\,(g: B \to A) = 3^3 = 27$$



Cases when $g(x) \neq f(x)$

for every $i \in \{B\}$, 2 *choices for* $g(i)$: *the two numbers that are different from* $f(i)$ *i.e.* $g(i) \neq f(i)$

$\therefore$ *No. of functions* $f(i): g(x) \neq f(x)$ is $2^3 = 8$

$\therefore$ *No. of functions* $f(i): g(x) = f(x)$ is $2^3 = 27 - 8 = 19$

$\therefore k = 19 - 10 = 9$

Q 7.

JEE Mathematics – Functions

Let $f:\{1,2,3\} \rightarrow \{1,2,3\}$ *be a function. If the number of functions* $g:\{1,2,3\} \rightarrow \{1,2,3\}$ *such that* $f(x) = g(x)$ *for atleast one* $x \in \{1,2,3\}$ *is* k*, then* $(k-10)$ *is equal to*

f(1)	1	1	2	1	3	1		f(10)	1	2	2	1	3	1		f(19)	1	3	2	1	3	1
f(2)	1	1	2	1	3	2		f(11)	1	2	2	1	3	2		f(20)	1	3	2	1	3	2
f(3)	1	1	2	1	3	3		f(12)	1	2	2	1	3	3		f(21)	1	3	2	1	3	3
f(4)	1	1	2	2	3	1		f(13)	1	2	2	2	3	1		f(22)	1	3	2	2	3	1
f(5)	1	1	2	2	3	2		f(14)	1	2	2	2	3	2		f(23)	1	3	2	2	3	2
f(6)	1	1	2	2	3	3		f(15)	1	2	2	2	3	3		f(24)	1	3	2	2	3	3
f(7)	1	1	2	3	3	1		f(16)	1	2	2	3	3	1		f(25)	1	3	2	3	3	1
f(8)	1	1	2	3	3	2		f(17)	1	2	2	3	3	2		f(26)	1	3	2	3	3	2
f(9)	1	1	2	3	3	3		f(18)	1	2	2	3	3	3		f(27)	1	3	2	3	3	3

$f(1): \{(1,1),(2,1),(3,1)\}$
$f(2): \{(1,1),(2,1),(3,2)\}$
$f(3): \{(1,1),(2,1),(3,3)\}$
$f(4): \{(1,1),(2,2),(3,1)\}$
$f(5): \{(1,1),(2,2),(3,2)\}$
$f(6): \{(1,1),(2,2),(3,3)\}$
$f(7): \{(1,1),(2,3),(3,1)\}$
$f(8): \{(1,1),(2,3),(3,2)\}$
$f(9): \{(1,1),(2,3),(3,3)\}$

$f(1): \{(1,2),(2,1),(3,1)\}$
$f(2): \{(1,2),(2,1),(3,2)\}$
$f(3): \{(1,2),(2,1),(3,3)\}$
$f(4): \{(1,2),(2,2),(3,1)\}$
$f(5): \{(1,2),(2,2),(3,2)\}$
$f(6): \{(1,2),(2,2),(3,3)\}$
$f(7): \{(1,2),(2,3),(3,1)\}$
$f(8): \{(1,2),(2,3),(3,2)\}$
$f(9): \{(1,2),(2,3),(3,3)\}$

$f(1): \{(1,3),(2,1),(3,1)\}$
$f(2): \{(1,3),(2,1),(3,2)\}$
$f(3): \{(1,3),(2,1),(3,3)\}$
$f(4): \{(1,3),(2,2),(3,1)\}$
$f(5): \{(1,3),(2,2),(3,2)\}$
$f(6): \{(1,3),(2,2),(3,3)\}$
$f(7): \{(1,3),(2,3),(3,1)\}$
$f(8): \{(1,3),(2,3),(3,2)\}$
$f(9): \{(1,3),(2,3),(3,3)\}$

Q 8. JEE Mathematics – Functions

Let $f:(-2,2) \to (-2,2)$ be a continuous function such that $f(x) = f(x^2)\ \forall x \in d_f$

and $f(0) = \frac{1}{2}$, then the value of $4f\left(\frac{1}{4}\right)$ is equal to

Option 1: 1

Option 2: 2

Option 3: 3

Option 4: $\frac{1}{2}$

$$f:(-2,2) \to (-2,2)$$

$$f\,(0) = f(0^2) = \frac{1}{2}$$

$$f\,(1) = f(1^2) = f(-1)$$

$$f\left(\frac{1}{2}\right) = f\left(\frac{1}{4}\right) = f\left(-\frac{1}{2}\right)$$

$\therefore f(i)$ is a constant function $= \frac{1}{2}$

$$\therefore 4f\left(\frac{1}{4}\right) = 4\left(\frac{1}{2}\right) = 2$$

Q 9.

JEE Mathematics – Functions

Let $A = \{x_1, x_2, x_3, x_4, x_5\}, B = \{y_1, y_2, y_3, y_4\}$. *A function f is defined from A to B, such that* $f(x_1) = y_1$ *and* $f(x_2) = y_2$. *If the number of onto functions from A to B is n, then* $(n - 10)$ *is*

Option 1: 12

Option 2: 16

Option 3: 8

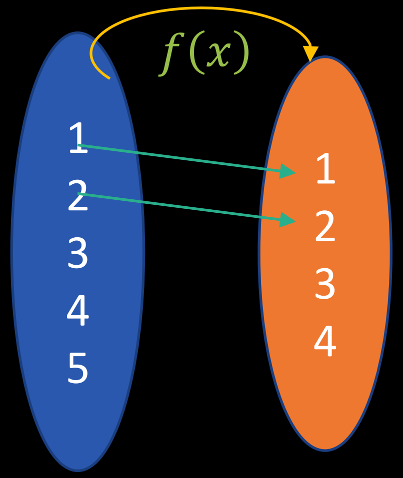
Option 4: 6

Onto function Range = codomain B $\rightarrow n(R) - 4$

$$^3C_2\ 2! = \frac{3(2)}{1(2)}(2)(1) = 6$$

$$^3C_2\ 2!\ 2 = \frac{3(2)}{1(2)}(2)(1)2 = 12$$

Total number of functions $= 6 + 12 = 18$



any 2 numbers from $\{3,4,5\}$ *and any 2 from* $\{3, 4\}$

Q 10.

JEE Mathematics – Functions

The area enclosed by the curve $|x+y-1|+|2x+y-1|=1$ *in sq. units is*

Option 1: 2   *Option 2:* 3   *Option 3:* 6   *Option 4:* 7

If both are positive, tthen $x+y-1+2x+y-1=1 \rightarrow 3x+2y=3$

$\rightarrow \frac{x}{1}+\frac{y}{\frac{3}{2}}=1$

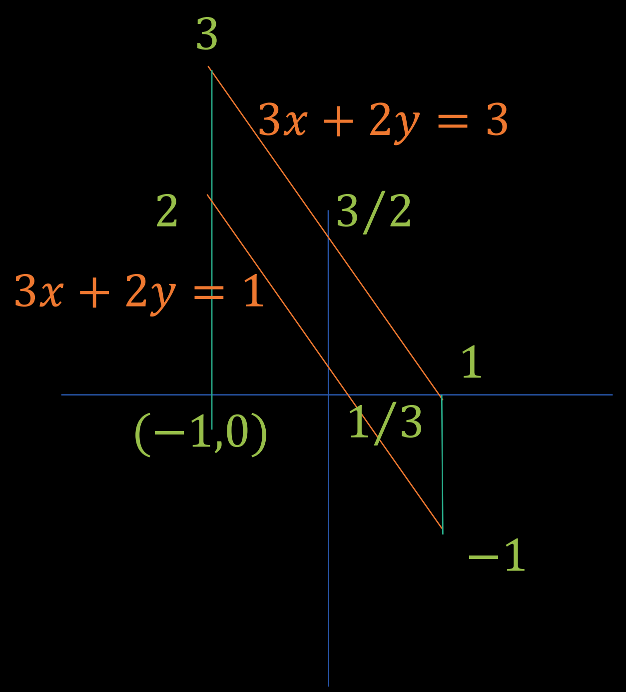
If $2x+y-1$ *is negative, tthen* $(x+y-1)-(2x+y-1)=1$

$\rightarrow x=1$

If $x+y-1$ *is negative, tthen* $-(x+y-1)+(2x+y-1)=1$

$\rightarrow x=-1$

If both are negative, tthen $-(x+y-1)-(2x+y-1)=1$ $\rightarrow \frac{x}{\frac{1}{3}}+\frac{y}{\frac{1}{2}}=1$



Required area $=\left(\frac{1}{2}(2)3\right)-\left(\frac{1}{2}\left(\frac{4}{3}\right)2\right)+\left(\frac{1}{2}(1)\frac{2}{3}\right)=2$ *sq. units*

Q 11.

JEE Mathematics – Functions

Consider set $A = \{x_1, x_2, x_3, x_4, x_5\}; B = \{y_1, y_2, y_3\}$. *Function f defined from A to B.*
Find the number of onto functions from A to B such that $f(x_1) = y_1$.

Option 1: 12

Option 2: 16

Option 3: 8

Option 4: 6

$A = \{x_1, x_2, x_3, x_4, x_5\}; B = \{y_1, y_2, y_3\}$

case 1: *If* 2 *elements go to* y_1 *from* $\{x_2, x_3, x_4, x_5\}$

$$No.of\ functions:\ n_1 = \frac{4!}{2!\,1!\,1!\,2!}3! = 36$$

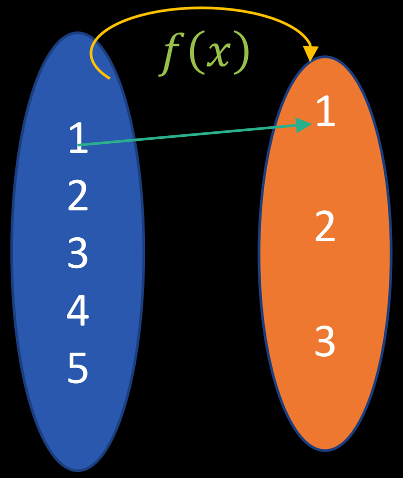
case 2: *If* 2 *elements go to* y_2 *and others go to* y_3

$$No.of\ functions:\ n_2 = \frac{4!}{2!\,21!\,2!}2! = 6$$

case 3: *If* 3 *elements go to* y_3 *and one goes to* y_3

$$No.of\ functions:\ n_3 = \frac{4!}{3!\,1!}2! = 8$$

$$Total\ no..of\ functions: n = 36 + 6 + 8 = 50$$



Q 12.

JEE Mathematics – Functions

Consider set $A = \{x_1, x_2, x_3, x_4, x_5\}; B = \{y_1, y_2, y_3\}$. *Function f defined from A to B.*
Find the number of functions from A to B such that $f(x_1) = y_1$ *and* $f(x_2) \neq y_2$.

$$A = \{x_1, x_2, x_3, x_4, x_5\}; B = \{y_1, y_2, y_3\}$$

for x_1, *there is only one option.*

for x_2, *there is only two options.* y_1 *or* y_3

for x_3, *there is three options.* y_1 , y_2 *or* y_3

for x_4, *there is three options.* y_1 , y_2 *or* y_3

for x_5, *there is three options.* y_1 , y_2 *or* y_3

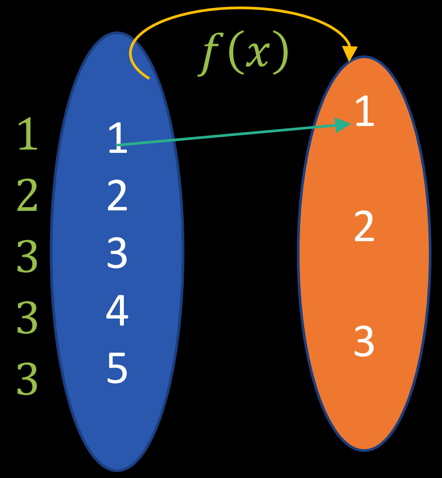
Total no.. of functions: $n = 1(2)(3)(3)(3) = 54$

Option 1: 64

[*Option 2:* 54]

Option 3: 27

Option 4: 81



Sets, Relations and Functions

Summary

We have learnt

Sets and their representation

Union, Intersection, Complement

Power set

Relation, types of relations, equivalence relations

Functions, One − one, into, onto functions

Composition of functions

Thank you

Dr. RK